The Pragmatist Roots and Some Expressivist Extensions of *The Dialogical Roots of Deduction*

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Dutilh Novaes's new book is both original and important.[1] *The Dialogical Roots of Deduction* investigates the relations between deduction and dialogue. Its approach is comprehensive, progressing along four interlocking, mutually-supporting dimensions: historical, philosophical, psychological, and in connection with mathematical practice. By doing that it substantially illuminates a number of distinctive features of deductive logical relations that philosophers of logic have found problematic or puzzling. These include the necessary *truth-preservingness* of deductive consequence relations, the irrelevance of the issue of whether or not one believes the premises and conclusions of deductive consequence relations, the distinctive sort of *perspicuousness* afforded by the possibility of unpacking deductive arguments into step-by-step chains, each of whose individual links is immediately cogent, and the nature of the *normative significance* of logical relations. There are substantial contemporary literatures devoted to each of these topics. But they are typically treated in isolation from one another. Perhaps the most impressive feature of the book, marking it as a landmark achievement in the field, is the fact that Dutilh Novaes offers a *systematic*, *unified* account that traces *all* of these phenomena back to the *same* source and persuasively explains them all on the *same* basis: the relation between deductive logical relations and dialogic practices.

I think it is particularly worthwhile to get clear about the nature of the central, weight-bearing relation between deduction and dialogue that Dutilh Novaes uncovers and elaborates by means of the metaphor of "roots." I want to begin by making a suggestion about how we might characterize *one* fundamental philosophical idea that animates this metaphor, in the hopes of clarifying its philosophical significance by connecting it to some other ideas. The idea I follow up on is that Dutilh Novaes shows us (among much else), how to understand relations of deductive consequence (what is expressed by the turnstile) in terms of dialogic practices. Then I want to consider one way of following out the clues suggested by that formulation, so as to generalize Dutilh Novaes ideas by applying them to areas beyond those in which she introduces them: from thinking about our peculiar and rarified deductive practices to thinking about reasoning in general.

[1] Catarina Dutilh Novaes, *The Dialogical Roots of Deduction: Historical, Cognitive, and Philosophical Perspectives on Reasoning* [Cambridge University Press, 2021].

I. Reason Relations and Reasoning Practices

Lewis Carroll's fable "Achilles and the Tortoise" vividly teaches us to distinguish, in John Stuart Mill's terms, "premises from which to reason" (including those codifying implication relations) from "rules in accordance with which to reason." He shows, in particular, that action-governing norms in the form of rules cannot be eliminated wholesale in favor of premises in the form of conditionals, on the pain of rational impotence. Gil Harman has radicalized this point, arguing for the initially astonishing conclusion that there are no such things as deductive rules of inference. If there were, presumably a paradigmatic one would would correspond to *modus ponens*, and would say something like: If you believe that *p* and you believe that *if p then q*, then you should believe that *q*. But, he points out, that would be a terrible general norm governing what one should *do*, inferentially, given those beliefs. For your collateral beliefs might give you much better reasons *against q* than you have *for* either *p* or *if p then q*. In that case, it seems, what one would have reason to *do* is to give up one of those commitments.

This argument points to the conclusion that we should distinguish between deductive logical *relations* of implication and incompatibility and inferential *activities* or *practices*. What logical relations establish deductively is that *p*, *if p then q*, and *not-q* are incompatible, because *p* and *if p then q* stand in the relation of *logical implication* to *q*, and *q* and *not-q* stand in the relation of *logical incompatibility* to each other. These deductive logical relations normatively *constrain* our reasoning activities. For they tell us that if we find ourselves committed to all of *p*, *if p then q*, and *not-q*, that we are in a normatively bad position. Those commitments are incompatible, we cannot be entitled to all of them. This entails a normative obligation to *do something*, to alter this normatively unsatisfactory situation. That *p* and *if p then q* imply *q* tells us that *in some sense, p* and *if p then q* together provide reasons *for q*. And that *not-q* is incompatible with *q* tells us that *in some sense not-q* provides a reason *against q*. But those reason *relations* do not *determine*, but only *constrain* what we are obliged to *do*, the *reasoning practices* or inferential activities of *changing* our commitments (and so entitlements) that we should engage in.

Deductive reason relations tell us about consequential relations among commitments, and about which commitments we can jointly be entitled to. The sense in which implication relations express what commitments provide reasons *for* what other commitments is something like that one is committed to the consequences of one's other commitments. And the sense in which incompatibility relations express what commitments provide reasons *against* what other commitments is something like that one is not entitled to commitments incompatible with one's

other commitments. But the implicit norms that one should acknowledge the consequences of one's commitments and not undertake commitments to which one is not entitled can collide. When those norms do collide, the reason relations by themselves don't dictate what interlocutors should *do*. They do not provide definitive guidance for reason*ing*: making inferences that alter one's commitments and entitlements.

Dutilh Novaes discusses the question of how to understand the normative significance of deductive logical relations (and the large literature that has grown up around this topic downstream from Harman's initial delineation of it, and in a much more sharply focused fashion in the wake of MacFarlane's pathbreaking formulations of it) as one among a battery of issues in contemporary philosophy of logic she addresses. I would like to foreground this topic, and consider how her project looks if we use it as a lens through which to view the whole thing. Looking at it from this perspective highlights in particular two of the master-ideas articulated and developed in the book.

1. We should understand the nature and significance of the deductive reason *relations* traditionally studied under the heading of "deductive logic" in terms of the role those relations of logical consequence and incompatibility play in *practices* of reason*ing*.
2. Those reasoning practices should be understood as essentially *dialogical*: as practices of giving and assessing reasons, by defending and challenging commitments.

Building on her work over the past decades, Dutilh Novaes argues that the way of thinking about logic encapsulated in these two claims not only has a long history, but has substantial claims to be the *traditional* way of understanding the nature of logic. This traditional insight was obscured in the twentieth century by formalist models of uninterpreted calculi thought of as illuminating mathematical proofs, understood either as themselves formal objects, or at best as monological proof-procedures. It is part of Dutilh Novaes's argument that the latter picture was never true to actual mathematical practice.

Let me say a bit more about each of these big ideas, putting them in my terms, rather than hers. The first claim is an overarching methodological commitment that orients the entire project. It is what the image of "roots" in the title is metaphorical for. I understand it in terms of a broadly *expressivist* view about logic: the point of logic is to make explicit essential structural features of reasoning. Further, I think of it as pursuing a broadly *pragmatist* approach to logic. By this I mean that the order of explanation appeals first to *pragmatics*, thought of as the theory of the *use* of logical expressions (the practices in which it *does* matter what interlocutors are committed or entitled to, and how those statuses change), in order then to understand the deductive logical relations expressed by applying those expressions (for which questions of belief or entitlement are "bracketed").

I think it is interesting to consider this pragmatist order of explanation in connection with an *inferentialist* understanding of the meanings or conceptual contents of logical locutions. Inferentialists about logic think of that the meaning of logical connectives consist in the role they play in deductive logical relations. In natural deduction formulations, the difference between the meanings of conditionals, negation, and disjunction in classical logic and intuitionistic logic, for instance, are taken by inferentialists to consist in the difference between the pairs of introduction and elimination rules that introduce and define those connectives. Inferentialism about the meaning of logical vocabulary (the content of logical concepts) is a much more widely shared and, arguably, much more plausible thesis than general semantic inferentialism that aims to extend the inferential-role account of the meanings of logical vocabulary to empirical descriptive vocabulary and beyond. (A good account both of the logical inferentialist species and the semantic inferentialist genus, see Jaroslav Peregrin's masterful recent book *Inferentialism*.[2] One of the points he makes there is that although the origins of enthusiasm for logical inferentialism are to be found in those who prefer to think about logic in the proof-theoretic terms due originally to Gentzen rather than the model-theoretic terms due originally to Tarski, inferentialists can think of proof-theory and model-theory just as rival metalanguages in which to specify inferential roles.)

A weak, minimal sort of logical inferentialism claims that the meanings of logical locutions are articulated by the deductive logical relations compound sentences containing them stand in to one another. In the context of such a view, Dutilh Novaes first master-idea shows up as a form of *pragmatism* about logic. By 'pragmatism' here I mean an order of explanation that runs from *pragmatics*, the study of the *use* of linguistic expressions, to *semantics*, the study of the *meaning* of that vocabulary, the conceptual contents its use expresses. Logical inferentialism supplies the middle term connecting pragmatics with semantics in this sort of pragmatism about logic, by understanding the meaning or conceptual content expressed by logical vocabulary in terms of the role that vocabulary plays in the relations of logical consequence and incompatibility, which are in turn to be explained by appeal to the use of the vocabulary in practices of reasoning.

Dutilh Novaes's second master-idea is worked out in the form of a regimented dialogical pragmatics of reasoning that she calls the "Prover-Skeptic" model of dialogue. This model is structured by two complementary functional social roles. The job of the proponent is rationally to defend a claim, by giving reasons for it, and the job of the skeptic is rationally to prove and

[2] [Palgrave Macmillan, 2014]

challenge it, by trying out reasons against it. One of Dutilh Novaes's central claims is the striking observation that just because these in some sense *antagonistic* roles are *complementary* in the particular way they are, they can also be seen as aspects of a fundamentally cooperative endeavor: examining the credentials of the arguments they contest.

Dutilh Novaes introduces her deeply historically grounded pragmatic model by arguing for its superiority over the earlier regimentations of dialogues by Lorenzen and Hintikka. And her principal concern is to show how a battery of properties of logical consequence relations that the philosophy of logic tradition has found to be both central and perhaps essential to its understanding of logical consequence relations and also deeply puzzling are illuminated by looking at the role they play in dialogic explorations of the credentials of claims that accord with the regimented Prover-Skeptic model. Her attention is accordingly focused on the historical antecedents of the model (looking backward) and its explanatory benefits (looking forward). I want to raise and begin to explore a different sort of issue: how to understand why things *must* be as she shows them to be. This is to look in a certain sense at the roots of the roots, or, maybe better, the ground that supports the roots she had brought into view for us.

The Prover-Skeptic pragmatics is motivated by the idea that the reasoning practices on the basis of which we are to understand deductive logical relations (and so, I want to say—though she does not—the semantic contents of logical concepts) have a distinctive *dialogical* structure. They are practices of making claims and assessing the *reasons* for those claims. Assessing the *rational* credentials of claims essentially involves doing two different kinds of thing: defending claims by giving *reasons for* them and challenging claims by giving *reasons against* them. Giving a reason *for* a claim is asserting something that *implies* it or has it as a consequence. Giving a reason *against* a claim is asserting something that is *incompatible* with it or rules it out. Accordingly, the two fundamental reason relations of implication and incompatibility can be understood in terms of the roles they play in these two fundamental kinds of acts of reason-giving.

In Dutilh Novaes's Prover-Skeptic dialogic model of reasoning practices, those roles are complementary and interdependent. That does seem to reflect, in a suitably idealized and regimented way, features of our actual practices. And it does exhibit pragmatic roots, in our practices of giving and challenging reasons, for the basic relations that our logics codify: logical consequence and logical consistency. It thereby provides a satisfactory and illuminating answer to the question Harman raised, about how we should understand the relations between deductive logical *relations* and inferential *practices*.

Further, I think we can see some considerations that might be appealed to in what amounts to a transcendental argument for the necessity of two generic roles of which Dutlh Novaes's Prover and Skeptic are more specific versions. As I have characterized them above, these are the roles of *defending* a commitment by giving reasons *for* it, and *challenging* a commitment by giving reasons *against* it.[3] In order to think further about these roles—to think, as it were, about the (meta)role played in reasoning practices by these two roles interlocutors can play—we might consider alternatives. We could call reasoning practices that consisted exclusively of offering reasons *for* "dogmatic" reasoning practices, reasoning practices that consisted exclusively of offering reasons *against* "skeptical" reasoning practices, and reasoning practices that allow both "critical" reasoning practices. The Prover-Skeptic model regiments critical reasoning practices. What would be wrong with its impoverished merely dogmatic and merely skeptical cousins?

We can think of *inquiry* as what practically pursues the issue of whether to accept or reject a commitment. Inquiry is *rational* insofar as it consists of assessing *reasons* for adopting one of these practical attitudes of acceptance or rejection. Citing reasons is what *entitles* one to accept or reject a commitment. (In the whole picture, within a properly critical practice, the commitments will be properly understood as *propositionally contentful*, as *claimables*, insofar as they stand in reason relations of implication and (in)consistency with other claimables, in virtue of the role they play in critical practices of assessing their rational credentials.) Dogmatic reasoning practices, then, admit only the offering of reasons *for* commitments, that is, reasons to *accept* claims. In a wonderful essay called "Why 'Not'?", Huw Price considers the practical deficiencies of what I am calling "dogmatic" reason-giving practices.[4] He imagines "ideological positivists," who do not have a way of denying or rejecting a claim. They lack any practical acknowledgment of the *incompatibility* of two claims. (It will follow that in their logic they have no way of *negating* a claim—hence the issue of his title.) He illustrates why such practices wouldn't work with a nice dialogue:

> Me: 'Fred is in the kitchen.' (Sets off for kitchen.)
> You: 'Wait! Fred is in the garden.'
> Me: 'I see. But he is in the kitchen, so I'll go there.' (Sets off.)
> You: 'You lack understanding. The kitchen is Fred-free.'

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[3] For the sake of my argument, I am abstracting away from the details of Dutilh Novaes's model in a way that might be thought to falsify the characterization. In particular, I am ignoring the Skeptic's role as granting or failing to grant premises, and as objecting to moves specifically by offering counterexamples.

[4] Huw Price "Why 'Not'?" Mind, New Series, Vol. 99, No. 394 (Apr., 1990), pp. 221-238. Published by: Oxford University Press on behalf of the Mind Association. Stable URL: https://www.jstor.org/stable/2254961. Dialogue quoted is from p. 224.

Me: 'Is it really? But Fred's in it, and that's the important thing.'
(Leaves for kitchen.)

Unless our commitments can *exclude* some other commitments, *preclude* our *entitlement* to those other commitments, they can't guide our actions. We would learn *nothing* practically from finding out that someone is entitled to a commitment—say, "Fred is in the garden,"—unless it meant that one could *not* be entitled to some other commitments—"Fred is in the kitchen,"—which accordingly count as *incompatible* with it. The very same reason that is a reason *for* one commitment must also be a reason *against* some others. The very same reason entitling one to *accept* a commitment must be a reason obliging one to *reject* those others, in the sense of *precluding entitlement* to them.

This parity of reasons for and reasons against, the requirement that reasons to accept one commitment *must be* reasons to reject some others, is acknowledged in the Scholastic slogan adopted and exploited by Spinoza, "*omnis determinatio est negatio.*" And it is at the heart of modern information theory, in the form of its founder Claude Shannon's idea that the information conveyed by a signal can be measured by the extent to which it *rules out* practical options on the part of the recipient compared to the situation before receipt of the signal. In monologic conceptions of mathematical proof, this fundamental oppositional dimension of reasoning is represented by the felt obligation to show the *consistency* of the conclusions one has derived, the theorems one has proven from the axioms, given the rules. Showing consistency is precisely showing that none of the conclusions one has arrived at *contradict* any of the others, in the sense that commitment to one precludes entitlement to the other. In this way the role of the Skeptic in Prover-Skeptic dialogues is internalized into the monological procedure. But this essential feature of the reasoning is also *hidden* in the monological conception of deductive practice, left *implicit* in the requirement of consistency of theories. It is made explicit by the dialogic division of rational labor in the Prover-Skeptic model.

Appreciating the transcendental practical necessity of incorporating reasons against (reasons to reject) in dogmatic reasoning practices also points us to the considerations that let us dispose pretty quickly of the notional possibility of purely skeptical reasoning practices: those that would admit only reasons against, reasons to reject commitments. Dogmatic practices practically erase the distinction between commitments interlocutors are entitled to and those they are not entitled to by treating commitments as promiscuously compatible. Skeptical practices practically erase the distinction between commitments interlocutors are entitled to and those they are not entitled to by offering no way at all for interlocutors to become entitled to their commitments. We could imagine that these skeptical reasoning practices take place against a background where interlocutors count as entitled to whatever commitments they make by

default. Then the practices are Popperian: the practice of reasoning is the process of falsifying commitments, unmasking them by showing we are *not* entitled to them. Presumably, the reasons against are *also* ones that are "innocent until proven guilty," in that those who offer such reasons count as entitled to them until and unless *those* entitlements are undercut by contrary assertions. Here, too, though, it is hard to see that it makes any difference at all which commitments interlocutors are entitled to. There is no way to transmit such entitlements to further commitments. The *only* use one can make of default entitlements that have not yet been infirmed is to infirm other commitments. Perhaps the practice is that discredited commitments lose their capacity to discredit others. But so what? *Any* incompatible claim can be made to attack a given commitment, and so long as it has not yet been discredited by contrary assertion, it will count as disconfirmatory.

Reasoning is essentially about the rational *credentials* of possible commitments, which is to say about *entitlements* to those commitments. It cannot consist entirely of obligatory movements from commitments to consequential commitments. Reasons must include reasons entitling one to accept or to reject further commitments. Reasoning exploits relations linking entitlement to one commitment to entitlement to others. (That is why the "roots" of relations of logical consequence and (in)consistency are to be found in reasoning practices.) Entitlement to a commitment must be consequential for entitlements to other commitments; it must be transmissible. And it must entitle one *both* to *accept* some further commitment *and* to *reject* some other commitments. Dogmatic or skeptical reasoning practices either grow entitlements wildly and without limit, or shrink them wildly and without limit. Only critical reasoning practices can flourish. To use a Hegelian image, both the inhalation and the exhalation of reasoning are essential to its life and health. The stylized roles of Prover and Skeptic, Proponent and Opponent, in Dutilh Novaes's model of reasoning are accordingly idealizations distilling features essential to any practices recognizable as *reasoning*. Ultimately, they reflect the essential dual role played by the choice of accepting or rejecting commitments, including practical commitments to *do* something. Reasons transmit entitlements to those attitudes, and so essentially, and not just accidentally come in the two flavors of reasons for and reasons against commitments. One central contribution of Dutilh Novaes's book is its working out of the pragmatist idea that logical reason *relations* of implication and inconsistency are to be understood in terms of the role they play in *practices* with this essentially dual structure, which is internal to the idea of reasoning as such.

II. Material Reason Relations and Substructurality

Notice that everything I said in the previous discussion of the transcendental pragmatic reasons for the Prover-Skeptic model to have the structure it does were addressed to practices of giving and challenging *reasons* generally. There was no restriction to practices of offering *logically* good reasons to accept or reject claims. I want not only to offer further grounding for Dutilh Novaes's ideas about the dialogic pragmatic roots of deductive logical reason relations (which she might well think is not needed), but also to suggest how those ideas might be developed further (albeit in directions she might well not be willing to take them).

I want to argue that at least one of Dutilh Novaes's lines of thought can be extended from specifically *logical* reason-relations and their rootedness in dialogic practice to reason-relations *generally*. And, as a consequence, I want to extend that idea from reason-relations that meet the traditional Tarski-Gentzen structural conditions of monotonicity and transitivity to reason-relations that are radically *sub*structural, in the sense that the do *not* satisfy those structural conditions. And I want to claim that it is a signal indication of the depth and power of the "rootedness in dialogue" idea that it *does* continue to apply in this radically extended domain.

Relations of logical consequence depend essentially on the occurrence of specifically *logical* vocabulary in the premises and conclusions. (Logical inferentialists believe that that is because such relations articulate the conceptual content expressed by logical vocabulary. But I am claiming only something weaker here.) We can see that, because logically good implications remain good upon arbitrary substitution of nonlogical for nonlogical vocabulary (and logical contradictions also remain logical contradictions under such substitutions). This has led many to conclude that deductive reason relations hold in virtue of the logical form of the sentences involved. It is one of innovations and signal advantages of Dutilh Novaes's account that she does *not* take its *formality* to be a defining characteristic of deductive logical relations of consequence and inconsistency. Her view is that

> Rather than being that in virtue of which an argument is deductively valid, logical forms/schemata are in fact convenient devices that allow us to track deductive validity with less effort (though for a limited range of arguments). [*DRD* 18]

In this way, many false trails and needless complications are avoided.[5]

[5] John MacFarlane's 2000 Pittsburgh Ph.D. dissertation "What Does It Mean to Say that Logic is Formal?" is a conceptually wide-ranging and historically informed discussion of the many ramifications of the issue of the formality of logic that has been vastly and justly influential (in spite of never having been officially published).

One need not take on that traditional commitment to deductive reason relations holding in virtue of their logical form, however, in order to accept the weaker claim that those relations depend essentially on the occurrence of logical vocabulary in the claimables they relate, in the sense of their being substitutionally robust in the way just described. And that observation opens the way to the realization that we can start with the observation that there least appear to be implications and inconsistencies that essentially depend in a corresponding way on the occurrence of *non*logical vocabulary.

Pittsburgh is to the south of Montreal.
therefore
Montreal is to the north of Pittsburgh.

is an implication that depends essentially on the contents of the concepts expressed by the nonlogical terms "south" and "north," and is robust under arbitrary substitution for the terms "Pittsburgh" and "Montreal." And the fact that the set of claims

{Monochromatic surface A is green, Monochromatic surface A is red}

is inconsistent essentially depends on the contents of the concepts expressed by the nonlogical terms "green" and "red" and "monochromatic," and is robust under substitutions for the place-holder "A." Wilfrid Sellars calls these "material" implications and inconsistencies. The term should be thought of as contrasting with "logical." (Sellars probably picked it because he *did* think of logic as essentially formal. But acknowledging the contrast his terminology marks does not depend on that collateral commitment.)

If we now ask how reason relations of consequence or implication and inconsistency or incompatibility that essentially depend on the occurrence of specifically *logical* vocabulary (in the sense defined by the method of noting invariance under substitution) should be thought of as standing to reason relations of implication and incompatibility that essentially depend on the occurrence of *non*logical vocabulary, another traditional philosophical thesis about logic comes into view. This is *logicism* about reason relations generally. It claims that *all* good reasons are ultimately to be understood as *logically* good reasons. What it is for some claims *rationally* to imply another, or to be *rationally* incompatible with it, is for them *logically* to imply or be *logically* inconsistent with it. The substantial commitment that is fundamental to this sort of approach is what Sellars calls

> ...the received dogma...that the inference which finds its expression in "It is raining, therefore the streets will be wet" is an enthymeme.[6]

[6] Sellars "Inference and Meaning," reprinted in *Pure Pragmatics and Possible Worlds* J. Sicha (ed.) [Ridgeview Publishing Co. 1980] (hereafter, *PPPW*), pp. 261/313.

According to this line of thought, wherever an inference is endorsed, it is because of belief in a conditional. Thus the instanced inference is understood as implicitly involving the conditional "If it is raining, then the streets will be wet". With that "suppressed" premise supplied, the inference is an instance of the formally valid scheme of conditional detachment.

Logicism about reasons generally is a very strong claim. For it is committed to the open-ended program of reconstructing all good reasons as logically good. It faces well-known difficulties even on its most friendly ground of mathematical reasoning. But the logicist program faces even larger challenges in addressing the reason relations that show up for instance in practical reasoning (such as jurisprudential reasoning), inductive scientific or probabilistic Bayesian reasoning, and the sorts of implication and inconsistency appealed to in informal conversation. Further, logicism mut understand any and every grasp of relations of being a reason for or reason against as an essentially *logical* ability. The grasp of logic attributed must be implicit, since it need not manifest itself in any capacity to complete or fill in “enthymematic” reasoning, manipulate specifically logical vocabulary, assess logical derivations, or distinguish logical tautologies. It is hard to see how such an occult logical ability can be specified in terms sufficiently non-question-begging to do any actual explanatory work.

Dutilh Novaes is not committed to logicism about reasons generally. Her topic is restricted to deductive logical reasons. And acknowledging that relations of rational implication and incompatibility can depend essentially on the presence of logical vocabulary *or* the presence of nonlogical vocabulary in no way requires making the additional reductionist logicist claim about the relations between these two cases. We can simply observe that in addition to logical consequence and inconsistency there are *material* relations of implication and incompatibility. Implications and incompatibilities can depend essentially on the occurrence of particular bits of vocabulary in them, as assessed by substitutional tests. Some of that vocabulary is logical, and its occurrence is essential to the logical goodness of reasons for and against, and some is not, and its occurrence is essential to the material goodness of reasons for and against.

Note that a question that remains even after we have adopted this relaxed, nonlogicist attitude toward the relations between logically good reasons and materially good reasons (understood in terms of their ranges of substitutional robustness) concerns the distinction between logical and nonlogical vocabulary, on which it depends. This is the *demarcation* issue in the philosophy of logic: how to distinguish specifically logical vocabulary, or the concepts it expresses. We logical expressivists, who are also semantic inferentialists, take it that logical vocabulary is distinguished by playing a distinctive expressive role. That role is to make explicit the reason relations of consequence and incompatibility that articulate the contents of *all*

concepts—beginning with the *material* relations of consequence and incompatibility that articulate the contents expressed by *non*logical vocabulary. The expressive job characteristic of conditionals is to make implication relations explicit in the (logically extended) object language, and the expressive job characteristic of negation is to make incompatibility relations explicit in the object language. Dutilh Novaes does not explicitly address the demarcation question. In practice, she adopts a historical-developmental approach, rather than a strictly functional one.

However it is with logical consequence, *material* consequence relations do not in general satisfy the strong structural conditions that have traditionally been thought to be essential features of specifically *logical* consequence. In particular, as Dutilh Novaes acknowledges, ordinary reasoning, by contrast to deductive reasoning, is often nonmonotonic. Adding further premises can turn good implications into bad ones. As a result, ordinary reasoning admits the construction of Sobel sequences, where the addition of further considerations flips the polarity of implications in both directions:

- If I strike this dry, well-made match, then it will light.
- If I strike this dry, well-made match in a strong magnetic field, then it will not light.
- If I strike this dry, well-made match in a strong magnetic field but inside a Faraday cage, then it will light.
- If I strike this dry, well-made match in a strong magnetic field but inside a Faraday cage, and in a room from which the air has been evacuated, then it will not light.
- …

  And so on.

One *might* insist that all the implications codified in these conditionals are strictly false, expressing enthymematic approximations of the true conditionals that would explicitly include as premises *all* the potentially defeating or enabling conditions. (I would caution that *if* one takes such a line, one should *not* do so because of an implicit commitment to logicism about the goodness of material implications.) But speaking against such a conception is the suspicion that there is no definite totality of such defeating and enabling conditions, or that if there were, it would in any case not be finitely statable.

Even rigorous reasoning in mature sciences depends for its cogency on tacit assumptions it would be at least tedious and possibly simply impossible to state explicitly. When I apply a bit of ideal theory, say Ohm's law relating current, voltage, and resistance, in order to make predictions about what various meter-readings will be if I make an intervention in an electrical circuit, any inferences I make will be defeasible by a host of potential confounding collateral circumstances, such as all the sorts of defects there could be in the measuring apparatus. Medical diagnosis consists largely of making inferences from history and physical findings that

are then found to be defeated by conditions revealed by further tests. (This is the plot of every episode of medical shows such as "House.") And legal reasoning in trials, both civil and criminal, depends essentially on the making of rebuttable presumptions and the drawing of rebuttable conclusions.

The features of these practices that acknowledge in-principle defeasibility by further auxiliary hypotheses serving as collateral premises are not avoidable conveniences of reason-giving practices. They stem from the necessity for finitely statable arguments in the face of infinite possibilities for collateral information that would infirm the implications in question. Our empirical reasoning cannot avoid what is explicitly acknowledged by the use of *ceteris paribus* clauses. It is not that appending such a clause to an implication magically turns a defeasible implication into an indefeasible one. (Latin phrases whose utterance can make that sort of difference are called "spells.") The expressive function of *ceteris paribus* clauses is just to acknowledge explicitly the defeasibility of an implication. (On pain of triviality, it can't be that its force is "*q* follows from *p*, except in cases where it doesn't.")

III. Substructural Material Reason Relations Support Well-Behaved Logics

Pointing out the substructural character of material implication (and incompatibility) relations risks severing the connection with logic entirely. What reason is there to think of them as relations of *rational consequence* at all. After all, Tarski had good reason to think his structural conditions were minimal conditions on consequence *tout court* We can imagine Tarski or Gentzen saying: When we say that these are *reason* relations, relations of implication, consequence, or rational following from, the best evidence for *our* claim is that you can do *logic* with them. Even if you don't think that doing is *all* there is to reasoning (for instance, because you think being able to engage in dialogic reasoning practices is important, too), still, we can argue that it is a *necessary* condition of being *reason* relations that they articulate a *logic*.

And highlighting their substructurality, the failures of monotonicity (and, although I have not gone into it here, even transitivity) makes it looks as though *material* consequence relations have nothing to do with logic at all. It is one thing to object to reason relations such as material consequence relations being *reducible* to logical reason relations; that is to reject logicism. It would be a *much* stronger claim that a relation is intelligible as being a *rational* consequence relation if it has *nothing* to do with, no principled relation to, logic and *logical* consequence relations.

The way I would like to put the challenge ("as the one playing the role of Skeptic says to the one playing the role of Prover") is this: The claim that substructural material relations of, as it were, implication and incompatibility really qualify as *reason* relations, in the sense of underwriting relations of being a reason *for* and being a reason *against*, depends on, has as a necessary condition, standing in the right relation to *logic*. To assess the claim in the light of that challenge, we need to settle what the "right relation to logic" is. I have already rejected the logicist reading of it, which fixes the logic side in advance, and then treats as rational, as *reason* relations, only what can be reconstructed in terms of specifically *logical* relations of deductive consequence and inconsistency. And I suspect that there is no way of answering the question of what the relation is that holds between logic and the reason relations that codify what is a reason for or against what that is neutral across widely varying philosophies of logic. So I will address it from the point of view of what I take to be the correct answer to the question.

Logical expressivists are in a way the mirror-image of logicists about how to understand the relations between logic and the relations of being a reason for and being a reason against. Where logicists think logic *determines* what premises provide reasons for a claim and what premises

provide reasons against it, we logical expressivists think the expressive role distinctive of logical vocabulary is to let us *say* what premises provide reasons for a claim and what premises provide reasons against it. We *start* with reason relations, and introduce logical vocabulary to *express* them. Conditionals let us express implication relations in the form of claimables that can both serve as and stand in need of reasons and so be rationally supported and challenged. And negation does the same thing for relations of incompatibility.

So for us expressivists, the question of whether substructural material relations of consequence and incompatibility qualify as genuine *reason* relations in virtue of their relation to logic (admittedly not the only consideration that bears on the larger question of being reason relations) comes down to the question of whether they are codifiable in logical terms in a way that is both formally tractable and recognizably similar to traditional logics. To this question we can respond with a resounding "Yes." Recent work by Ulf Hlobil and Dan Kaplan, in our research group "Research on Logical Expressivism" (ROLE) has shown how to build well-behaved logics on top of substructural relations of material implication and incompatibility.[7]

The idea is to begin with what we call a "material semantic frame" (MSF) defined on a language L_0 consisting of a finite set of logical atoms. Such a frame consists of a consequence relation $|\sim_0$, and a distinguished set of sets of atomic sentences that are treated as incoherent. There are both single-succedent and multi-succedent sequent calculus versions of the logic, but I'll start by talking just about the single-succedent case. Then we can encode the material incoherence of a set $\Gamma \subseteq L_0$ as a sequent: $\Gamma|\sim_0 \bot$.

We impose only two minimal structural conditions on the base MSF: contexted reflexivity or containment (CO) and a principle we call "*ex falso fixo quodlibet*" (ExFF). The first says that for any set of sentences Γ and any sentence A in L_0,

CO: $\Gamma, A |\sim_0 A$.

A is a material consequence of any set of premises that contains A. The second is a version of explosion or *ex falso quodlibet* adapted for a nonmonotonic setting. It can happen that although Γ is incoherent, it is defeasibly so, in that adding some further sentences to it yields a coherent set. We mark only the *indefeasibly* or persistently incoherent sets by requiring that

ExFF: $\forall(A \in L_0)\ [\forall(\Delta \subseteq L_0)\ \Gamma,\Delta |\sim_0 \bot \Rightarrow \Gamma,\Delta |\sim_0 A]$.

Persistently incoherent premise-sets materially imply everything.

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[7] Ulf Hlobil, "A Nonmonotonic Sequent Calculus for Inferentialist Expressivists," [In P. Arazim & M. Danˇcák (Eds.), The Logica Yearbook 2015 (pp. 87–105). College Publications.] and Daniel Kaplan, "A Multi-Succedent Sequent Calculus for Logical Expressivists," [In P. Arazim & M. Danˇcák (Eds.), The Logica Yearbook 2017. College Publications.].

We do *not* require that material semantic frames have consequence relations or incoherence properties that are monotonic. That is, we do *not* require:
MO: $\forall(A\in L_0)\forall(\Gamma,\Delta\subseteq L_0)\ [\ \Gamma\ |\sim_0 A\ \Rightarrow \Gamma,\Delta\ |\sim_0 A]$.
And we do *not* require that the consequence relations be transitive. That is, we do *not* require that Cut holds:
CT: $\forall(A,B\in L_0)\forall(\Gamma,\Delta\subseteq L_0)\ [\ (\Gamma\ |\sim_0 A\ \&\ \Gamma,A\ |\sim_0 B) \Rightarrow \Gamma\ |\sim_0 B]$.

We can extend the atomic base language L_0 in the usual way, by adding sentential logical connectives to produce a language consisting of logically complex sentences formed by applying those connectives recursively to the language L_0 of logical atoms. Hlobil and Kaplan show how to use Gentzen-style sequent calculus connective rules to introduce specify the consequence and incompatibility reason relations (relations of being a reason for and being a reason against) of (sets of) sentences in the logically extended language.

As expressivists about the functional roles that demarcate specifically *logical* vocabulary, we want to impose two crucial restrictions on the connective rules defining conditionals and negations. Since we want conditionals to codify implication relations (including material ones), we want the conditional operator to satisfy the Ramsey condition, in both directions:
Ramsey Condition: $\Gamma|\sim A\rightarrow B$ **iff** $\Gamma,A|\sim B$.
That is, a premise-set implies a conditional just in case the result of adding the antecedent to that premise-set implies the consequent. A conditional that satisfies this equivalence can be called a "Ramsey-test conditional," since Frank Ramsey first proposed thinking of conditionals this way.

Since we want negation to codify incompatibility relations (including material ones), we want the negation operator to satisfy the Minimal Negation condition, in both directions:
Minimal Negation Condition: $\Gamma|\sim\neg A$ **iff** $\Gamma,A|\sim\bot$.
That is, a premise-set implies not-A just in case A is incompatible with that premise-set. (It follows that $\neg A$ is the minimal incompatible of A, in the sense of being implied by everything that is incompatible with A.)

Underwriting these *bi*conditionals requires connective definitions that are reversible. Gentzen's student Ketonen showed how to reformulate Gentzen's connective definitions so as to make them reversible. We adopt his formulations. It turns out that in order to extend the underlying MSF governing logically atomic sentences conservatively, we need to mix and match additive and multiplicative rules for conjunction and disjunction. (Otherwise, monotonicity gets built in.) In spite of those distinctions, our connective definitions, like Ketonen's originals, are fully equivalent to Gentzen's, in the sense that there is a derivation of a conclusion from some set

of axioms using Ketonen's connective definitions just in case there is derivation of that conclusion from the same set of axioms—in their system, always instances of Reflexivity: A|~A (we use Contexted Reflexivity or Containment, CO)—using Gentzen's connective definitions.[8]

In this way, new consequence and incompatibility relations are defined for the logically extended language by deriving them from more basic, nonlogical relations of consequence and incompatibility. Because the extension is conservative over the underlying material semantic frame—in that the implications and incompatibilities involving only logically atomic sentences are the same in the extension as in the original—the new reason relations will not in general be monotonic or transitive. Nonetheless, the result yields traditional intuitionistic and classical logics as limiting cases.

This is so in three different ways.

- First, the resulting system is supraclassical (in the multisuccedent case) or supraintuitionistic (in the single-succedent case)—that is, they validate all the classical or intuitionistic implications, respectively.
- Second, they yield straightforwardly classical and intuitionistic consequence relations (in the multisuccedent and single-succedent cases, respectively) if the MSFs they extend are "flat"—that is, consist exclusively of instances of Contexted Reflexivity or Containment, of the form Γ, A|~A.
- Third, the purely *logical* portion of the implication and incompatibility relations defined over the logically extended language are fully structural, and indeed, are just the classical and intuitionist relations of consequence and inconsistency. By "purely logical" I mean the consequences that hold upon arbitrary substitution of nonlogical for nonlogical vocabulary. In our systems, these are the same consequences and incompatibilities that hold no matter what underlying material semantic frame one extends logically.

The substructural character of prelogical relations of consequence and incompatibility accordingly presents no bar to codifying them logically: using conditionals to express implications and negation to express incompatibility. And the logics that result are both well-behaved and familiar. So someone who, while rejecting the logicists' identification of *all* reason relations with *logical* reason relations—someone who thinks that *good* reasons for and against need not be exclusively *logically* good reasons for and against—nonetheless thinks both that

[8] The Ketonen rules we use are "mixed": rules with two top sequents are additive and rules with a single top sequent are multiplicative. This is required to avoid the connective rules forcing structural monotonicity in the extended language. It is very close to the system called 'G3cp' by Negri, Von Plato, and Ranta (2008), but with material axioms.

logic provides paradigmatic reason relations and that logic stands in some special and distinctive relation to reason relations generally has no reason to take the nonmonotonicity and nontransitivity of material relations of consequence and incompatibility to entail that they are not genuine *reason* relations. The fact that logical relations of being a reason for and being a reason against satisfy strong structural constraints does not speak against the substructural relations that articulate the contents of ordinary, nonlogical concepts being genuine reason relations. One can even do logic with them.

IV. Dialogues and Substructural Material Reason Relations

The argument I have just rehearsed addressed potential objections to acknowledging nonlogical, material relations of consequence and incompatibility as genuine reason relations, as being rational relations of supporting and ruling out conclusions, on the basis that they are unlike *logical* reason relations in being in general nonmonotonic and nontransitive. I did not consider the crucial new set of considerations that the *Dialogical Roots of Deduction* has put on the table. I addressed only a traditional requirement on being genuine reason relations**:** that one be able to do logic with them. Dutilh Novaes digs deeper. For she has an account of *why* you can do logic with genuine reason relations: because of the roles they play in dialogic practices of defending claims by giving reasons for them and challenging claims by giving reasons against them. What qualifies something as a reason relation is the functional role they play in dialogic practices of reason*ing*. Reason *relations* are to be understood in terms of reasoning *practices* having the dialogic form regimented in the Prover-Skeptic model.

I understand the book as saying (again, among *much* else) that if you want to understand what the turnstile expressing logical consequence *means*, what you are *saying* when you say that A is a deductive consequence of Γ, you have to look to the role that reason relation plays in dialogic practices of reasoning—practices that, according to the Prover-Skeptic model, are practices of defending claims by giving reasons *for* them and challenging claims by giving reasons *against* them.

According to this line of thought, the question we should be asking is whether the substructural character of material, nonlogical relations of consequences and incompatibility prevents them from playing a proper role in suitable analogues of Prover-Skeptic dialogues. If and insofar as they can play a functional role in such dialogues that is recognizably the same as that of fully structural, logical relations of deductive consequence (and inconsistency), they will qualify as genuine reason relations.

To determine the answer to this question, my colleague, Pitt doctoral student Yao Fan wrote a Python program to implement Prover-Skeptic dialogues based on substructural material semantic frames. We looked only at dialogues involving giving reasons for and against (defending and challenging) claims within the logically atomic, base language. After all, we know how what happens at that ground level completely determines what happens in the language that has been extended by the introduction of logical vocabulary—including the fully structural relations of *logical* consequence and inconsistency that result.

For demonstration purposes, we work in an artificial language with only 7 sentences: {a_1, a_2, a_3, a_4, a_5, a_6}, often abbreviated just by their subscripts. We define an arbitrary substructural *material semantic frame* (MSF) by specifying a material relation of *consequence* between premise-sets and single-sentence conclusions and a property of material *incoherence* that characterizes some sets of sentences. A conclusion is treated as *incompatible* with a premise-set iff their union is incoherent. MSFs accordingly codify *reason relations* of implication and incompatibility that will function dialogically to define relations of being a reason *for* and being a reason *against*.

Appendix 1 displays a sample MSF defined on that language of 7 logical atoms. It has 190 significant material implications, listed at lines 9ff.. Breaking these out, if you look at lines 70 to 75, listing the premise-sets that imply sentence 2, you will see that although 1 implies 2 and 3 implies 2, {1,3} does not imply 2, though {1,3,4,5,6} does. The material consequence relation is accordingly nonmonotonic. The MSF contains 60 materially incoherent sets (out of the 2^7=128 possible subsets of the language). These, too, are nonmonotonic. So looking at lines 147 to 154, listing the premise-sets that are incompatible with sentence 2, 3 is incompatible with 2 and 4 is incompatible with 2, but {3,4} is not.[9]

Appendix 2 displays one sample dialogue conducted on the basis of a sample material semantic frame like the one in Appendix 1. (For some of our experiments, we run tens of thousands of dialogues based on each MSF.) The leftmost column, after the line numbers, is the turn number of the dialogue. This one is 45 steps long. The next column lists the participants responsible for each move. The names we have given to our versions of Dutilh Novaes's "Prover-Skeptic" are "Claimant" (CL) and "Critic" (CR). They alternate. The next column to the right indicates the move that is challenged or defended by the move currently being made. Next is the *pragmatic significance* of the move made. I will return to that when we walk through the dialogue. The next column lists the move. It is always a reason *for* a claim, marked with "entails," or a reason *against* a claim, marked with "excludes." The next column, marked CL_AC, lists what the *claimant* (CL) is *committed* to *accept* after the move in that row. The column to its right lists what the *claimant* is *committed* to *reject* after that move. The next two

[9] At the very beginning of Appendix 1 you will see a long code that makes this MSF recoverable (for repeated dialogic experiments). It gives some indication how many MSF meeting our minimal structural constraints there are, even for the very basic 7-member material base language of our toy example. The set of incoherent sets is an element of the powerset of the powerset of the language, which has 2^128 elements. Even removing permutations, that is more than 10^30 possibilities. And there are even more ways to pick the material implications.

columns record what the claimant is *entitled* to *accept* and *entitled* to reject. The last four columns give the same information for the *critic* (CR).

This dialogue begins with the claimant (CL) putting forward a *proposal*. The dialogue consists of an inquiry into the defensibility of that proposal, given the underlying MSF. In our example, the claimant offers a reason for proposition a_2. That reason for is a set of premises that materially imply a_2, according to the MSF. All moves in the dialogue, whether giving reasons defending a claim or reasons challenging a claim, are drawn from the governing base MSF, which is understood as the common semantic basis of the dialogue. Making that move commits the claimant CL to all the premises, and to the conclusion. And, in the absence—thus far—of any challenge, under our rules the claimant counts as at this point entitled by default to those commitments. (The dialogues proceed according to what in *Making It Explicit* I call a "default-and-challenge" structure of entitlement.)

There are two ways to challenge a reason offered in any move in the dialogue. One can offer a reason *against* one of the premises, or a reason challenging the conclusion. If the conclusion is an endorsement (what is challenged was a reason for, an implication, marked by "entails") this will be a reason *against* ("excluding") the conclusion. If the conclusion was itself a rejection (an "exclusion"), then one challenges that conclusion by offering a reason *for* it. These are the "premise challenges" and "conclusion challenges" listed in the column of pragmatic significances ("PragSig") of the moves. In our example, in Turn 1 the critic CR challenges one of the premises of the proposal, by offering a reason *against* a_4. Making this move commits and default entitles CR to accept the premise and to reject the conclusion of the challenging reason. It does not alter the claimant's commitments, but does remove entitlement to the challenged premise and the conclusion it supports, while leaving intact the default entitlement to the other premises of the proposal.

The dialogue proceeds by claimant CL challenging the conclusion of CR's premise challenge, offering a reason against it. Since CR offered a reason *against* premise a_4 of the proposal, CL responds by defending that premise, offering a reason *for* it. This removes CR's entitlement to the conclusion of the challenge in Turn 1. It expands the claims CL is committed to accept, adds new default entitlements to those commitments, and restores CL's entitlement to the challenged premise. The interlocutors can challenge or defend any of the previous moves, not just the immediately preceding move. Notice that at Turn 5, the critic CR abandons the argument over the proposal premise a_4 and mounts a new challenge directly to the original proposal, by offering a conclusion challenge to it: a reason *against* its conclusion a_2.

The dialogue continues in this way until one interlocutor can no longer find in the MSF a reason it is eligible to put forward, defending or attacking the proposal, given its current commitments to accept and reject claims. In this case, at the close of the dialogue, the claimant CL has not managed to sustain entitlement to the conclusion of the proposal, a_2. The proposal is accordingly defeated. This is the scorekeeping outcome of the competitive aspect of the dialogue: either the proposal is vindicated and CL wins, or it is defeated, and CR wins. However, the *point* of the dialogue, toward which the activity of both interlocutors is directed, is investigating the credentials of the proposal. One crucial scorekeeping expression of the cooperative aspect of the enterprise is that the interlocutors have established a significant *common ground*. At the end of the day, they are *both* committed *and entitled* to propositions a_0, a_1, a_3, and a_5. One of our interests in this project lies in investigating connections between features of the underlying MSF and the emergence of common ground in dialogues investigating the credentials, the defensibility, of different proposals. (And a second phase of the project, inspired by Girard's ludics, goes the other way around, deriving the reason relations codified in MSFs from dialogues.) But all that is a topic for another time.

The principal conclusion I want to draw here from our experiments is that they *show* that substructural material semantic frames corresponding to material reason relations qualify as *reason* relations in the "dialogically rooted" sense: they play the right role in dialogic reasoning practices. The fact that they are nonmonotonic and nontransitive does not in any way disqualify them from supporting dialogues cooperatively-competitively investigating the credentials of claims by defending them with reasons and challenging them with reasons. I understand Dutilh Novaes as saying that if you want to understand what the turnstile expressing logical consequence *means*, what you are *saying* when you say that A is a deductive consequence of Γ, you have to look to the role that reason relation (plus inconsistency, I want to say) plays in dialogic practices of reasoning, practices that, according to the Prover-Skeptic model are practices of defending claims by giving reasons for them and challenging claims by giving reasons against them.

I propose that Dutilh Novaes's articulation of a tight connection between dialogic reasoning practices, having the Prover-Skeptic structure of giving and assessing reasons, on the one hand, with the paradigmatic reason relations (implication and incompatibility) of classical deductive logic, argues for the intelligibility of radically substructural *material* reason relations of implication and incompatibility, on the basis that such relations support dialogic reasoning practice that have the Prover-Skeptic structure of giving and assessing reasons. Further, I take it that she has explicitly left room for some such consequence of her views. She says, for instance:

> I submit that a plethora of kinds of dialogues should be embraced, and insofar as different logical systems will correspond to different kinds of dialogues, we end up with different, equally legitimate logical systems. What defines which logic is the 'right one' are the motivations of participants when engaging in a given dialogical situation, and their mutual agreement in terms of the structural and logical features of that particular conversation.

and

> Rather than entailing an overly permissive 'anything goes' attitude, the dialogical perspective in fact allows for the formulation of restrictions on what can count as a legitimate logical system: one that corresponds to a plausible kind of dialogue that people may actually feel compelled to engage in (though admittedly much work remains to be done on specific criteria of adequacy for dialogical systems).[10]

[10] Both passages from section 2.4 of Chapter Four of *DRD*.

V. Conclusion

The aim of this essay has been to place the achievement of the *Dialogic Roots of Deduction* in a larger philosophical context. That wider context encompasses a number of claims. At their core is the idea that what I called "reason *relations*" of implication and incompatibility mediate between essentially dialogic reasoning *practices* and the introduction of specifically logical vocabulary, and so relations of deductive *logical* consequence and inconsistency. Implication relations are to be understood dialogically in terms of the functional role they play in providing reasons *for* claims, by appeal to which those claims can be rationally defended. Incompatibility relations are to be understood dialogically in terms of the functional role they play in providing reasons *against* claims, by appeal to which those claims can be rationally challenged. And on the basis of reason relations so understood in terms of their role in reasoning, we can introduce specifically *logical* vocabulary that lets us make those relations explicit in a logically extended object language. The connective definitions that perform that distinctive expressive job then underwrite specifically *logical* relations of consequence and inconsistency relating logically complex sentences. And those logical reason relations underwrite properly deductive proofs.

The superstructure of *logical* reason relations exhibits the classical Tarski-Gentzen structural features, including monotonicity and transitivity, even if the underlying *material* reason relations do not. The perhaps paradoxical claim is that by widening our perspective to include *pre*logical reasoning practices of defending claims by giving *non*logical reasons for them and challenging claims by giving *non*logical reasons against them, we can bring into relief some of the fine structure of the "rootedness" of logical deduction in dialogical practices, which is Dutilh Novaes's focal concern. For that more encompassing perspective makes visible the role of *material* relations of rational implication and incompatibility in the elaboration of logical relations of deductive consequence and inconsistency, on the one hand, and the role of those reason relations in practices of rational dialogue, on the other.

One might worry that telling the story the way I have here stands in tension with one of the central contentions of the book. This is the observation that the practice of deductive logical proof is the product of quite specific, contingent, culture-bound, historically conditioned circumstances. Developing this thesis is one of the buttresses of Dutilh Novaes's account of the dialogical roots of deduction. I do not think the story I have sketched contradicts or even threatens the book's insight as to the rarified, historically situated character of the process by which deductive logical proofs crystallize out of stylized dialogic reasoning practices. Rather, I

think the two perspectives are compatible and complementary. Engaging in logical deductive practices is indeed, as Dutilh Novaes teaches, a contingent, sophisticated, late-coming product of a quite specific tradition. But the logic that results from that tradition *expresses*, in its distinctive sophisticated, only contingently available way, fundamental features of reasoning in general—and so of discursive practices as such.

Appendix 1: A Toy Material Semantic Frame (MSF):

File - Dialogic Pragmatics_Main_August 7th

C:\Users\Bob\PycharmProjects\DP3\venv\Scripts\python.exe "C:/Users/Bob/Dropbox/NonMonCon/Spring 2019/Python/Dialogic Pragmatics_Main_August 7th.py"
You can retrieve this MSF using the Decode_MSF function with the fo11owing code:
len7imp3602002723205398433466171673337429638782917331628808079771009345429871437539567224914141885592298793025743415964344511408641175005374801328544741107053845223114297228769023447809451696603279707161376269659879390305466493695289231169881724371950273092460901928716127820inc3639474140251240480754444105219686 46
This MSF contains in total 658 implications, among which 190 are pragmatically significant, 448 are required by CO, 7 are required by ExFF and 13 are strange in the sense that the premises and the conclusion are jointly persistently inconsistent.
(Note that **if** an implication **is** required both by CO and ExFF, **it's** considered to be required by CO but not ExFF.)
This MSF contains 60 inconsistent sets, among which 6 are persistently inconsistent.

This MSF contains the following 190 pragmatically significant implications, i.e. implications that are not required by CO or ExFF and are not strange.
'{0, 1, 2, 3, 4}|~5', '{0, 1, 2, 3, 5}|~6', '{0, 1, 2, 3}|~6', '{0, 1, 2, 5}|~4', '{0, 1, 2, 6}|~3',
'{0, 1, 2}|~4', '{0, 1, 2}|~5', '{0, 1, 3, 4, 5}|~2', '{0, 1, 3, 4, 5}|~6', '{0, 1, 3, 4, 6}|~ 5',
'{0, 1, 3, 4}|~2', '{0, 1, 3, 4} |~5', '{0, 1, 3, 5, 6}|~2', '{0, 1, 3, 5, 6} |~4', '{0, 1, 3, 5}|~2',
'{0, 1, 3, 5}|~4', '{0, 1, 3, 5}|~6', '{0, 1, 3, 6}|~2', '{0, 1, 3, 6}|~4', '{0, 1, 3, 6}|~5',
'{0, 1, 3}|~5', '{0, 1, 3}|~6', '{0, 1, 4, 5, 6}|~3', '{0, 1, 4, 5}|~3', '{0, 1, 4, 5}|~6',
'{0, 1, 4, 6}|~5', '{0, 1, 4}|~3', '{0, 1, 5, 6}|~3', '{0, 1, 5, 6}|~4', '{0, 1, 5}|~2',
'{0, 1, 5}|~3', '{0, 1, 5}|~4', '{0, 1, 5}|~ 6', '{0, 1, 6}|~2', '{0, 1, 6}|~3',
'{0, 1, 6}|~4', '{0, 1, 6}|~5', '{0, 1}|~2', '{0, 1} 4', '{0, 1}|~6',
'{0, 2, 3, 4}|~5', '{0, 2, 3, 4}|~6', '{0, 2, 3, 5, 6}|~1', '{0, 2, 3, 5}|~1', '{0, 2, 3, 5}|~4',
'{0, 2, 3, 5}|~6', '{0, 2, 3, 6}|~ 1', '{0, 2, 3}|~1', '{0, 2, 3}|~5', '{0, 2, 3}|~6',
'{0, 2, 4, 5}|~1', '{0, 2, 4, 6}|~ 3', '{0, 2, 4}|~5', '{0, 2, 4}|~6', '{0, 2, 5, 6}|~1',
'{0, 2, 5}|~4', '{0, 2, 6}|~3', '{0, 2, 6}|~4', '{0, 3, 4, 5}|~2', '{0, 3, 4, 6}|~1', '{0, 3, 4, 6}|~2',
'{0, 3, 4, 6}|~5', '{0, 3, 5, 6}|~1', '{0, 3, 5, 6}|~2', '{0, 3, 5, 6}|~4', '{0, 3}|~2',
'{0, 3}|~ 6', '{0, 4, 5, 6}|~1', '{0, 4, 5, 6}|~3', '{0, 4, 5}|~3', '{0, 4, 5}|~ 6',
'{0, 4, 6}|~ 2', '{0, 4, 6}|~ 3', '{0, 4}|~ 1', '{0, 4}|~ 2', '{0, 4}|~3',
'{0, 4}|~6', '{0, 5, 6}|~2', '{0, 6}|~1', '{0, 6}|~4', '{0, 6}|~5',
'{0}|~2', '{0}|~3', '{1, 2, 3, 4, 5}|~6', '{1, 2, 3, 4}|~ 6', '{1, 2, 3, 5, 6}|~ 0',
'{1, 2, 3, 6}|~0', '{1, 2, 3, 6} |~4', '{1, 2, 3, 6} |~5', '{1, 2, 4, 5, 6}|~ 3',
'{1, 2, 4, 5}|~6', '{1, 2, 4, 6}|~ 5', '{1, 2, 4}|~3', '{1, 2, 5, 6}|~3', '{1, 2, 5}|~4',
'{1, 2}|~0', '{1, 2}|~ 6', '{1, 3, 4, 5, 6}|~2', '{1, 3, 4, 5}|~0', '{1, 3, 4, 5}|~6',
'{I, 3, 4, 6}|~0', '{1, 3, 4}|~0', '{I, 3, 4}|~5', '{I, 3, 5, 6}|~0', '{I, 3, 5, 6}|~4',
'{I, 3, 5}|~0', '{I, 3, 5}|~4', '{I, 3, 5}|~6', '{1, 3, 6}|~4', '{I, 3} |~0',
'{1, 3}|~ 5', '{1, 3} |~ 6', '{1, 4, 6}|~ 3', '{1, 4}|~0', '{1, 4}|~6',
'{1, 5, 6}|~ 4', '{1, 5}|~ 3', '{1, 5}|~ 4', '{1, 6}|~3', '{1, 6}|~5',
'{1}|~0', '{1}|~2', '{1}|~3', '{1}|~4', '{2, 3, 4, 5, 6}|~ 1',
'{2, 3, 4, 5}|~1', '{2, 3, 4, 6} |~', '{2, 3, 4, 6}|~ 5', '{2, 3, 4}|~1', '{2, 3, 4}|~5',
'{2, 3, 4}|~6', '{2, 3, 5, 6}|~1', '{2, 3, 5, 6}|~4', '{2, 3, 5}|~0', '{2, 3, 5}|~1',
'{2, 3, 5}|~4', '{2, 3, 5}|~6', '{2, 3, 6}|~0', '{2, 3, 6}|~1', '{2, 3, 6}|~5',
'{2, 3}|~4', '{2, 4, 5, 6}|~1', '{2, 4, 5}|~1', '{2, 4, 6}|~3', '{2, 4}|~1',
'{2, 4}|~6', '{2, 5, 6}|~0', '{2, 5, 6}|~3', '{2, 5}|~ 0', '{2, 5}|~3',
'{2, 5} |~4', '{2, 6}|~0', '{2, 6} |~3', '{2}|~3', '{2}|~4',
'{2}|~ 5', '{2}|~6', '{3, 4, 5, 6}|~ 1', '{3, 4, 5}|~0', '{3, 4, 5}|~6',
'{3, 4, 6}|~5', '{3, 4}|~2', '{3, 4}|~5', '{3, 4}|~6', '{3, 5}|~0',
'{3, 5}|~2', '{3, 5}|~4', '{3}|~2', '{4, 5, 6}|~1', '{4, 5, 6}|~2',
'{4, 5}|~3', '{4, 6}|~1', '{4, 6}|~3', '{4}|~0', '{4}|~1',
'{4}|~2', '{4}|~5', '{5, 6}|~1', '{5, 6}|~3', '{5}|~3',
'{5}|~6', '{6}|~0', '{6}|~3', '{6}|~4', '{6}|~5',

This MSF contains the following 13 implications that are strange in the sense that the premises and the conclusion are jointly persistently inconsistent. We currently do not allow agents to use these implications as reason-fors.
'{0, 1, 2, 3, 4, 5}|~6', '{0, 1, 2, 3, 4}|~ 6', '{0, 1, 2, 4}|~6', '{0, 1, 2, 5, 6}|~ 4', '{0, 1, 3, 4, 5, 6}|~2',
'{0, 1, 4, 5, 6}|~2', '{0, 2, 3, 4, 5}|~6', '{0, 2, 3, 5, 6} 4', '{|~, 2, 4, 6}|~1', {0, 3, 4, 5, 6}|~2',
'{I, 2, 3, 4, 5, 6}|~0', '{I, 2, 3, 4, 6}|~0', '{I, 2, 4, 5, 6}|~0'

Thus**, this MSF has the following pragmatically significant reason-fors**:

0 has the following 21 pragmatica11y significant reasons for itself:
'{1, 2, 3, 5, 6}', '{1, 2, 3, 6}', '{1, 2}', '{1, 3, 4, 5}', '{1, 3, 4, 6}',
'{1, 3, 4}', '{1, 3, 5, 6}', '{1, 3, 5}', '{1, 3}', '{1, 4}',
'{1}', '{2, 3, 4, 6}', '{2, 3, 5}', '{2, 3, 6}', '{2, 5, 6}',
'{2, 5}', '{2, 6}', '{3, 4, 5}', '{3, 5}', '{4}',
'{6}'

1 has the following 27 pragmatica11y significant reasons for itself:
'{0, 2, 3, 5, 6}', '{0, 2, 3, 5}', '{0, 2, 3, 6}', '{0, 2, 3}', '{0, 2, 4, 5}',
'{0, 2, 5, 6}', '{0, 2, 6}', '{0, 3, 4, 6}', '{0, 3, 5, 6}', '{0, 4, 5, 6}',
'{0, 4}', '{0, 6}', '{2, 3, 4, 5, 6}', '{2, 3, 4, 5}', '{2, 3, 4}',
'{2, 3, 5, 6}', '{2, 3, 5}', '{2, 3, 6}', '{2, 4, 5, 6}', '{2, 4, 5}',
'{2, 4}', '{3, 4, 5, 6}', '{3, **4}', '{4,** 5, 6}', **'{4,** 6}',
'{4}', '{5, 6}'

2 has the following 23 pragmatica11y significant reasons for itself:
'{0, 1, 3, 4, 5}', '{0, 1, 3, 4}', '{0, 1, 3, 5, 6}', '{0, 1, 3, 5}', '{0, 1, 3, 6}',
'{0, 1, 5}', '{0, 1, 6}', '{0, 1}', '{0, 3, 4, 5}', '{0, 3, 4, 6}',
'{0, 3, 5, 6}', '{0, 3}', '{0, 4, 6}', '{0, 4}', '{0, 5, 6}',
'{O}', '{1, 3, 4, 5, 6}', '{1}', '{3, 4}', '{3, 5}',
'{3}', '{4, 5, 6}', '{4}'

3 has the following 32 pragmatica11y significant reasons for itself:
'{0, 1, 2, 6}', '{0, 1, 4, 5, 6}', '{0, 1, 4, 5}', '{0, 1, 4}', '{0, 1, 5, 6}',
'{0, 1, 5}', '{0, 1, 6}', '{0, 2, 4, 6}', '{0, 2, 5, 6}', '{0, 2}',
'{0, 4, 5, 6}', '{0, 4, 5}', '{0, 4, 6}', '{0, 4}', '{0}',
'{I, 2, 4, 5, 6}', '{I, 2, 4}', '{I, 2, 5, 6}', '{I, 4, 6}', '{I, 5}',
'{I, 6}', '{1}', '{2, 4, 6}', '{2, 5, 6}', '{2, 5}',
'{2, 6}', '{2}', '{4, 5}', '{4, 6}', '{5, 6}',
'{5}', '{6}'

4 has the following 30 pragmatica11y significant reasons for itself:
'{0, 1, 2, 5}', '{0, 1, 2}', '{0, 1, 3, 5, 6}', '{0, 1, 3, 5}', '{0, 1, 3, 6}',
'{0, 1, 5, 6}', '{0, 1, 5}', '{0, 1, 6}', '{0, 1}', '{0, 2, 3, 5}',
'{0, 2, 6}', '{0, 2}', '{0, 3, 5, 6}', '{0, 6}', '{I, 2, 3, 6}',
'{I, 2, 5}', '{I, 3, 5, 6}', '{I, 3, 5}', '{I, 3, 6}', '{I, 5, 6}',
'{1, 5}', '{1}', '{2, 3, 5, 6}', '{2, 3, 5}', '{2, 3}',
'{2, 5}', '{2}', '{3, 5, 6}', '{3, 5}', '{6}'

5 has the following 26 pragmatica11y significant reasons for itself:
{0, 1, 2, 3, 4} . {0, 1, 2} . {0, 1, 3, 4, 6} . {0, 1, 3, 4} . {0, 1, 3, 6} .
'{0, 1, 3}', '{0, 1, 4, 6}', '{0, 1, 6}', '{0, 2, 3, 4}', '{0, 2, 3}',
'{0, 2, 4}', '{0, 2, 6}', '{0, 3, 4, 6}', '{0, 6}', '{1, 2, 3, 6}',
'{I, 2, 4, 6}', **'{I,** 3, 4}', **'{I,** 3}', **'{I,** 6}', '{2, 3, 4, 6}',
'{2, 3, 4}', '{2, 3, 6}', '{2}', '{3, 4, 6}', '{4}',
'{6}'

6 has the following 31 pragmatically significant reasons for itself:
'{0, 1, 2, 3, 5}', '{0, 1, 2, 3}', '{0, 1, 3, 4, 5}', '{0, 1, 3, 5}', '{0, 1, 3}',
'{0, 1, 4, 5}', '{0, 1, 5}', '{0, 1}', '{0, 2, 3, 4}', '{0, 2, 3, 5}',
'{0, 2, 3}', '{0, 2, 4}', '{0, 2, 5}', '{0, 3}', '{0, 4, 5}',
'{0, 4}', '{1, 2, 3, 4, 5}', '{1, 2, 3, 4}', '{1, 2, 3}', '{1, 2, 4, 5}',
'{1, 2}', '{1, 3, 4, 5}', **'{1,** 3, 5}', '{1, 3}', '{1, **4}',**
'{2, 3, **4}',** '{2, 3, 5}', '{2, **4}',** '{2}', '{3, 4, 5}',
'{5}'

This MSF contains the following 60 inconsistent sets:
'{0, 1, 2, 3, 4, 5, 6}', '{0, 1, 2, 3, 4, 6}', '{0, 1, 2, 3, 4}', '{0, 1, 2, 3, 6}', '{0, 1, 2, 4, 5, 6}',
'{0, 1, 2, 4, 6}', '{0, 1, 2, 4}', '{0, 1, 2, 6}', '{0, 1, 3, 4, 6}', '{0, 1, 3, 4}',
'{0, 1, 3, 5}', '{0, 1, 3, 6}', '{0, 1, 3}', '{0, 1, 4, 5}', '{0, 1, 4, 6}',
'{0, 1, 5, 6}', '{0, 2, 3, 4, 5, 6}', '{0, 2, 3, 4, 5}', '{0, 2, 3, 5, 6}', '{0, 2, 3, 5}',
'{0, 2, 4, 5, 6}', '{0, 2, 4, 5}', '{0, 2, 4, 6}', '{0, 2, 5, 6}', '{0, 2}',
'{0, 3, 4, 5, 6}', '{0, 3, 4, 6}', '{0, 3, 4}', '{0, 3, 5}', '{0, 3, 6}',
'{0, 4, 6}', '{0, 5}', **'{I,** 2, 3, 4, 5}', **'{I,** 2, 3, 4, 6}', **'{I,** 2, 3, 5, 6}',
'{I, 2, 3, 6}', **'{I,** 2, 3}', **'{I,** 2, 4}', **'{I,** 2, 5}', **'{I,** 2, 6}',
'{1, 3, 4, 5, 6}', '{1, 3, 4, 6}', '{1, 3, 4}', '{1, 3, 5}', '{1, 3}',
'{1, 4}', '{2, 3, 4, 5, 6}', '{2, 3, 4, 5}', '{2, 3}', '{2, 4, 6}',
'{2, 4}', '{2, 5}', '{3, 4, 5, 6}', '{3, 4, 6}', '{3, 4}',
'{3, 5, 6}', '{4, 5, 6}', '{4, 5}', '{4, 6}', '{5, 6}'

Among all inconsistent sets, the following 6 are persistently inconsistent:
['{0, 1, 2, 3, 4, 5, 6}', '{0, 1, 2, 3, 4, 6}', '{0, 1, 2, 4, 5, 6}', '{0, 1, 2, 4, 6}', '{0, 2, 3, 4, 5, 6}', '{0, 2, 4, 5, 6}']

Thus, **this MSF contains the following reasons against**:

0 has the following 32 reasons against itself:
{1, 2, 3, 4, 5, 6} . {1, 2, 3, 4, 6} . {1, 2, 3, 4} . {1, 2, 3, 6} .
'{1, 2, 4, 6}', '{1, 2, 4}', '{1, 2, 6}', '{1, 3, 4, 6}', '{1, 3, 4}',
'{1, 3, 5}', '{1, 3, 6}', '{1, 3}', '{1, 4, 5}', '{1, 4, 6}',
'{1, 5, 6}', '{2, 3, 4, 5, 6}', '{2, 3, 4, 5}', '{2, 3, 5, 6}', '{2, 3, 5}',
'{2, 4, 5, 6}', '{2, 4, 5}', '{2, 4, 6}', '{2, 5, 6}', '{2}',
'{3, 4, 5, 6}', '{3, 4, 6}', '{3, 4}', '{3, 5}', '{3, 6}',
'{4, 6}', '{5}'

1 has the following 28 reasons against itself:
'{0, 2, 3, 4, 6}', '{0, 2, 3, 4}', '{0, 2, 3, 6}', '{0, 2, 4, 6}', '{0, 2, 4}',
'{0, 2, 6}', '{0, 3, 4, 6}', '{0, 3, 4}', '{0, 3, 5}', '{0, 3, 6}',
'{0, 3}', '{0, 4, 5}', '{0, 4, 6}', '{0, 5, 6}', '{2, 3, 4, 5}',
'{2, 3, 4, 6}', '{2, 3, 5, 6}', '{2, 3, 6}', '{2, 3}', '{2, 4}',
'{2, 5}', '{2, 6}', '{3, 4, 5, 6}', '{3, 4, 6}', '{3, 4}',
'{3, 5}', '{3}', '{4}'

2 has the following 31 reasons against itself:
'{0, 1, 3, 4, 5, 6}', '{0, 1, 3, 4, 6}', '{0, 1, 3, 4}', '{0, 1, 3, 6}', '{0, 1, 4, 5, 6}',
'{0, 1, 4, 6}', '{0, 1, 4}', '{0, 1, 6}', '{0, 3, 4, 5, 6}', '{0, 3, 4, 5}',
'{0, 3, 5, 6}', '{0, 3, 5}', '{0, 4, 5, 6}', '{0, 4, 5}', '{0, 4, 6}',
'{0, 5, 6}', '{0}', '{I, 3, 4, 5}', '{I, 3, 4, 6}', '{1, 3, 5, 6}',
'{I, 3, 6}', '{I, 3}', '{I, 4}', '{I, 5}', '{I, 6}',
'{3, 4, 5, 6}', '{3, 4, 5}', '{3}', '{4, 6}', '{4}',
'{5}'

3 has the following 32 reasons against itself:
'{0, 1, 2, 4}', '{0, 1, 2, 6}', '{0, 1, 4, 6}', '{0, 1, 4}', '{0, 1, 5}',
'{0, 1, 6}', '{0, 1}', '{0, 2, 4, 5}', '{0, 2, 5, 6}', '{0, 2, 5}',
'{0, 4, 5, 6}', '{0, 4, 6}', '{0, 4}', '{0, 5}', '{0, 6}',
'{1, 2, 4, 5}', '{1, 2, 4, 6}', '{1, 2, 5, 6}', '{1, 2, 6}', '{1, 2}',
'{1, 4, 5, 6}', '{1, 4, 6}', '{1, 4}', '{1, 5}', '{1}',
'{2, 4, 5, 6}', '{2, 4, 5}', '{2}', '{4, 5, 6}', '{4, 6}',
'{4}', '{5, 6}'

4 has the following 36 reasons against itself:
'{0, 1, 2, 3, 5, 6}', '{0, 1, 2, 3, 6}', '{0, 1, 2, 3}', '{0, 1, 2, 5, 6}', '{0, 1, 2, 6}',
'{0, 1, 2}', '{0, 1, 3, 6}', '{0, 1, 3}', '{0, 1, 5}', '{0, 1, 6}',
'{0, 2, 3, 5, 6}', '{0, 2, 3, 5}', '{0, 2, 5, 6}', '{0, 2, 5}', '{0, 2, 6}',
'{0, 3, 5, 6}', '{0, 3, 6}', '{0, 3}', '{0, 6}', '{1, 2, 3, 5}',
'{1, 2, 3, 6}', '{1, 2}', '{1, 3, 5, 6}', '{1, 3, 6}', '{1, 3}',
'{1}', '{2, 3, 5, 6}', '{2, 3, 5}', '{2, 6}', '{2}',
'{3, 5, 6}', '{3, 6}', '{3}', '{5, 6}', '{5}',
'{6}'

5 has the following 26 reasons against itself:
'{0, 1, 3}', '{0, 1, 4}', '{0, 1, 6}', '{0, 2, 3, 4, 6}', '{0, 2, 3, 4}',
'{0, 2, 3, 6}', '{0, 2, 3}', '{0, 2, 4, 6}', '{0, 2, 4}', '{0, 2, 6}',
'{0, 3, 4, 6}', '{0, 3}', '{0}', '{1, 2, 3, 4}', '{1, 2, 3, 6}',
'{I, 2}', '{I, 3, 4, 6}', '{I, 3}', '{2, 3, 4, 6}', '{2, 3, 4}',
'{2}', '{3, 4, 6}', '{3, 6}', '{4, 6}', '{4}',
'{6}'

A Sample Dialogue

TurnNum	Agent	TargetNum	PragSig	Move	CL_AC	CL_RC	CL_AE	CL_RE	CR_AC	CR_RC	CR_AE	CR_RE
0	CL	None	proposal	['a_0', 'a_3', 'a_4', 'a_5'] entails a_2	[0, 2, 3, 4, 5]	[]	[0, 2, 3, 4, 5]	[	[]	[]	[]	[]
1	CR	0	premise challenge	['a_6'] excludes a_4	[0, 2, 3, 4, 5]	[]	[0, 3, 5]	[	[6]	[4]	[6]	[4]
2	CL	1	conclusion challenge	['a_0', 'a_1', 'a_3', 'a_5'] entails a_4	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[6]	[4]	[6]	[]
3	CR	2	conclusion challenge	['a_3', 'a_5', 'a_6'] excludes a_4	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 5]	[	[3, 5, 6]	[4]	[3, 5, 6]	[4]
4	CL	3	conclusion challenge	['a_1', 'a_3', 'a_5'] entails a_4	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[3, 5, 6]	[4]	[3, 5, 6]	[]
5	CR	0	conclusion challenge	['a_0'] excludes a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 3, 5, 6]	[2, 4]	[0, 3, 5, 6]	[2]
6	CL	5	conclusion challenge	['a_1'] entails a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[0, 3, 5, 6]	[2, 4]	[0, 3, 5, 6]	[]
7	CR	6	conclusion challenge	['a_5'] excludes a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 3, 5, 6]	[2, 4]	[0, 3, 5, 6]	[2]
8	CL	7	conclusion challenge	['a_0', 'a_3'] entails a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[0, 3, 5, 6]	[2, 4]	[0, 3, 5, 6]	[]
9	CR	8	conclusion challenge	['a_0', 'a_5', 'a_6'] excludes a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 3, 5, 6]	[2, 4]	[0, 3, 5, 6]	[2]
10	CL	9	conclusion challenge	['a_4'] entails a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[0, 3, 5, 6]	[2, 4]	[0, 3, 5, 6]	[]
11	CR	10	conclusion challenge	['a_1', 'a_3', 'a_6'] excludes a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2]
12	CL	11	conclusion challenge	['a_0', 'a_1', 'a_3', 'a_5'] entails a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[]
13	CR	12	conclusion challenge	['a_3'] excludes a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2]
14	CL	13	conclusion challenge	['a_0', 'a_1'] entails a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[]
15	CR	14	conclusion challenge	['a_1', 'a_6'] excludes a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2]
15	CL	15	conclusion challenge	['a_3', 'a_5'] entails a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[]
16	CR	16	conclusion challenge	['a_1', 'a_5'] excludes a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2]
17	CL	17	conclusion challenge	['a_0', 'a_1', 'a_5'] entails a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[]
18	CR	18	conclusion challenge	['a_0', 'a_1', 'a_3', 'a_6'] excludes a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2]
19	CL	19	premise challenge	['a_0', 'a_1', 'a_4'] excludes a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[]
20	CR	20	premise challenge	['a_0', 'a_1', 'a_5'] excludes a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2, 4]
19	CL	19	premise challenge	['a_0', 'a_1', 'a_5'] excludes a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[4]
22	CR	22	conclusion challenge	['a_0', 'a_3'] entails a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2, 4]
19	CL	19	conclusion challenge	['a_0'] entails a_2	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[4]
24	CR	24	conclusion challenge	['a_1', 'a_3', 'a_5', 'a_6'] excludes a_2	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2, 4]
25	CL	25	premise challenge	['a_0', 'a_1', 'a_3'] excludes a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[4]
24	CR	24	conclusion challenge	['a_1', 'a_3'] excludes a_2	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[2, 4]
27	CL	27	conclusion challenge	['a_3'] entails a_2	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[4]
28	CR	28	conclusion challenge	['a_0', 'a_3', 'a_5'] excludes a_2	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[2, 4]
21	CL	21	conclusion challenge	['a_1'] entails a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 4, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[2]
26	CR	26	conclusion challenge	['a_0', 'a_1', 'a_3'] entails a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 4, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2]
29	CL	29	conclusion challenge	['a_3', 'a_4'] entails a_2	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[]
32	CR	32	premise challenge	['a_0', 'a_3', 'a_6'] excludes a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2, 4]
33	CL	33	premise challenge	['a_5'] excludes a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[]
34	CR	34	conclusion challenge	['a_0', 'a_1'] entails a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2, 4]
33	CL	33	conclusion challenge	['a_0', 'a_1', 'a_5'] entails a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[]
36	CR	36	conclusion challenge	['a_0', 'a_1', 'a_3'] excludes a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2, 4]
37	CL	37	conclusion challenge	['a_1', 'a_5'] entails a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[]
38	CR	38	conclusion challenge	['a_0', 'a_6'] excludes a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2, 4]
39	CL	39	premise challenge	['a_3', 'a_5'] excludes a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[]
38	CR	38	conclusion challenge	['a_0', 'a_3'] excludes a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[2, 4]
41	CL	41	conclusion challenge	['a_3', 'a_5'] entails a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[]
42	CR	42	conclusion challenge	['a_1'] excludes a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[2, 4]
43	CL	43	conclusion challenge	['a_0', 'a_1'] entails a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[]
45	CR	44	conclusion challenge	['a_5'] excludes a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[2, 4]

By the end of this stage, next player has the following 0 for-moves available:

By the end of this stage, next player has the following 0 against-moves available:

By the end of this stage, CL's proposed conclusion is rejected.

Process finished with exit code 0