

***Tales of the Mighty Dead:  
Historical Essays in the Metaphysics of Intentionality***

**Chapter Eight: Frege's Technical Concepts**

**I. Introduction**

Today we find ourselves at the outset of a golden age in the interpretation of Frege's philosophical writings. Judged by the number of articles, books, and seminars addressing his thought, interest in Frege is at an all-time high. More importantly, as Frege has come out of the shadow of Russell and Wittgenstein into the full light of critical attention, the degree of sophistication of discussion has achieved a quantum improvement. Many factors conspired to bring about this result, but two events may be singled out as having made special contributions both to the resurgence of interest in and to our greater understanding of Frege's work.

First is the publication, more than sixty years after his death, of that part of his *Nachgelassene Schriften* which survived the vicissitudes of the intervening years. These papers appeared in German in 1969 and in English in 1979.<sup>1</sup> Some of the contents are rough in form, though not without value. We are offered, for example, tables of contents and partial drafts of a textbook on logic and its philosophy which Frege made starts on at various crucial periods of his life. Even draft fragments of this sort permit important inferences from the order of presentation and different emphases given various topics to conclusions about the explanatory priorities Frege associated with his central technical concepts. But not all of the selections represent rough cuts or abandoned projects.

Included are some fully polished articles, dealing with Frege's most central technical concepts—fine examples of his concise, sometimes lapidary mathematician's prose—which he had tried unsuccessfully to publish. In a number of cases, these additional texts permit the resolution of exegetical disputes occasioned by what can now be seen to be accidental lacunae and merely apparent emphases in the canonical published corpus.

The other landmark event is the publication in 1973 of Dummett's monumental and long-awaited full-length treatment of Frege's philosophy of language.<sup>2</sup> It would be difficult to overestimate the significance of this classic work. Anyone interested in the interpretation of Frege must give it the same close attention owed to the primary texts. Its clarity of thought, patient rehearsal of considerations, and exercise of the best critical judgement in final appraisal will not be soon equalled. This essay will not offer a systematic account of Dummett's views, since the most important of these are so intimately tied up with the development of powerful novel approaches to contemporary philosophy of language as to defy brief characterization, even by their author. The original volume has now been supplemented with another containing many valuable amplifications and clarifications.<sup>3</sup> The result is a 1300 page corpus which, Dummett's complaints<sup>4</sup> to the contrary notwithstanding, by now deserves to be considered as setting out the canonical reading of Frege. It is so considered by the authors discussed below, and forms the background against which their own accounts are set out.

Two examples will serve to indicate the sort of interpretive advance signaled by these events. First, it was widely believed in the 1950's and 1960's that Frege did not intend the distinction between sense and reference to apply to functional expressions such as predicates, but only to complete expressions such as terms and sentences.<sup>5</sup> Although

the famous essay on sense and reference does not discuss such an application of that distinction, the *Nachlass* makes clear that this is only because that discussion was reserved for a further article which is quite explicit in its endorsement of that application, but which was repeatedly rejected for publication until Frege abandoned the attempt. Several other passages reprinted in [19] decisively refute the interpretation which would restrict the distinction to complete expressions. A somewhat less important mistake may also be mentioned as indicative, which was done in as much by Dummett's arguments as by the unearthing of further evidence. In 'On Sense and Reference' Frege says "One might also say that judgements are distinctions of parts within truth-values," and that "the reference of the word is part of the reference of the sentence."<sup>6</sup> These remarks have sparked the attribution of a variety of bizarre ontological views to Frege, centering on the notion of the True as representing the whole world, sometimes conceived as a Tractarian world of facts, sometimes as composed of objects (and what about the False?). The remarks stem from a hasty assimilation, soon explicitly rejected, of the relation between the argument of a function and the value it determines to the relation of part and whole. For although the function 'capital of. . . .' takes the value Stockholm when Sweden is taken as argument, Sweden is not part of Stockholm. Dummett's discussion of this issue has permanently disposed of the temptation to take these remarks seriously as interpretive constraints. We shall see below, however, that there remain genuine controversies which are not so easily disposed of (concerning the senses and referents of functional expressions) which may be regarded as successors to these two mistaken lines of thought.

Dummett has shown that Frege should be treated as a modern thinker in the sense that one can think about contemporary philosophical issues of considerable significance

by thinking about his concepts and their explanatory deployment, and that one cannot think about those concepts and their principles of deployment without thinking about such contemporary issues. In what follows those concepts are approached from three different directions. First, an attempt to interpret and develop Frege's technical scheme in light of contemporary discussions of the issues he was addressing is considered. Then attention is turned to an argument to the effect that ignoring the historical context in which Frege developed his theories, treating him we might say *merely* as a contemporary, leads to substantive misinterpretation of those theories. Finally, following one strand of the account of the path by which Frege developed and defended some of his central concepts, leads to a novel diagnosis of the status of those concepts.

## I

One important recent offering is David Bell's book *Frege's Theory of Judgment*.<sup>7</sup> This is a clear and well-written work. The issues it raises and the form in which they are addressed merit the attention of anyone interested in the significance for current inquiry of Frege's strategic deployment of a battery of technical concepts to explain various aspects of linguistic practice. Its title is worthy of some consideration. It is a measure of the degree of sophistication of contemporary Frege commentary that a controversy exists even over how one should describe the topic which his philosophical work addresses. Of course no one disputes his concern with the foundations of mathematical reasoning and knowledge, expressed above all in his three books, the *Begriffsschrift*, the *Grundlagen der Arithmetik*, and the *Grundgesetze der Arithmetik*. But the more general conceptual framework he found it necessary to elaborate in order to express clearly and precisely his claims about the nature of mathematics and its objects cannot easily be characterized

without prejudging substantial issues of interpretation. It may seem obvious that Frege was pursuing a project in the philosophy of language.<sup>8</sup> But such a description is misleading in the context of Frege's own insistence on the priority of thoughts (though not of thinkings) to their linguistic expression. For he was interested in natural languages only insofar as they permitted rough formulation of objective and language-independent thoughts, and he crafted artificial languages only as more adequate means for their expression. It would be inappropriate to build into the description of the subject-matter at the outset a post-Wittgensteinian conviction of the wrong-headedness of such an approach, by assimilating his concerns to contemporary investigations under the rubric "philosophy of language". One of the major theses of Sluga's book, discussed below, is that such Whiggish presuppositions of continuity of concern have consistently led Frege's readers to overlook important strands of his thought. Dummett has also suggested "theory of meaning" as a general characterization, but this seems to apply better to his own enterprise than to Frege's. For 'meaning' is correlative to 'understanding', and Frege's concern lay at least equally with reference, which is not in general grasped when one understands a claim, as with the sense which must be grasped in that case.

In his discussion of the book,<sup>9</sup> Dummett objects that Bell has misdescribed his topic, in that Frege's treatment of the act of asserting is the topic of only one chapter, while the rest of the book talks about the notions of sense and reference. This seems unfair, for the heading "theory of judgement" ought to entitle Bell to offer an account of the contents which are judged as well as of the acts which are the *judgings* of those contents. It has the advantage of placing Frege's concerns in appropriate historical and

philosophical context. Bell's denomination of Frege's topic as judgement displays his recognition of the importance Frege, in company with Kant and Wittgenstein, placed on inverting the traditional order of explanation which took concepts as primary and sought to account for judgments in terms of them. At least until 1891, Frege clearly regarded the claim that concepts can only be understood as the products of analysis of judgements as one of his most central insights. Although Bell does not say so, it is equally clear in the *Begriffsschrift* that Frege completes the inversion of the classical priority of concepts to judgements and judgements to syllogisms by taking the contents of sentences (judgement in the sense of what is judged rather than the judging of it) to be defined in terms of the inferences they are involved in.<sup>10</sup> Concepts are to be abstracted from such judgements by considering invariance of inferential role (which pertain only to judgments) under various substitutions for discriminable (possibly nonjudgemental) components of the judgement. Both in the introduction to *BGS* and in his essay on "Boole's logical Calculus and the BGS",<sup>11</sup> the virtue of the purely formal perspicuous language of inference in nonformal contexts is described as its permitting for the first time the scientific formation and expression of concepts. Although it is for this reason that Frege called his first work a "concept script", he later came to believe this phrase misleading precisely because it obscured his doctrine of the primacy of judgements. On the other hand, it would be equally misleading to describe Frege simply as a theorist of inference, in spite of the explanatory priority he accorded to it. For his primary theoretical focus always lay on the sentential and thence sub-sentential contents attributable to different expressions in virtue of the roles they played in inference, as revealed by their behavior under substitution. So "judgement", which is (a translation of) an expression Frege himself used pretheoretically

to describe the object of his theorizing, seems a good choice to delimit his subject matter. Like any other choice, however, it does prejudge some controversial issues of interpretation, for instance that concerning the persistence in Frege's thought of the so-called "context principle". It is often unclear exactly what this principle means, but the canonical statement of it is the *Grundlagen* claim that "only in the context of a sentence does a word have any significance". (I use 'significance' here for Frege's '*Bedeutung*' because in 1884 he had not yet distinguished *Sinn* from *Bedeutung*, and the undifferentiated term should be marked.) It is often claimed,<sup>12</sup> even by those such as Dummett who take the putative change in view to be a serious mistake, that when Frege achieved his mature views in 1891 with the formulation of that crucial distinction he discarded the context principle. If that is so, then Bell's choice of "theory of judgement" to describe the topic of the mature semantic views he discusses would be misleading or simply incorrect. As we shall see below, Sluga argues that Frege never relinquishes the context principle. Bell does not argue this, however, nor does he even claim it. He is simply silent on this issue, as on others concerning detailed questions about the attribution of various views to Frege based on textual evidence.

Bell's enterprise lies in a different direction entirely. He is concerned to look closely at the explanatory roles played by Frege's various concepts and at the ways in which Frege takes them to be related, in order to refine and reconstruct a broadly Fregean account of the nature of judgement. In keeping with this aim, he is not engaged in the exegesis of Fregean texts, and freely discards from his reconstruction a number of doctrines which Frege clearly held, in favor of incompatible principles (for instance, in Bell's reconstruction functional expressions are assigned senses but not referents). His

project is to salvage from Frege's account those insights which can be put together to form a workable theory of judgement. The result is broadly Fregean in endorsing the following "major strands" of Frege's theory:<sup>13</sup>

- I. There is the methodological principle that 'we can distinguish parts in the thought corresponding to the parts of a sentence, so that the structure of the sentence serves as a model of the structure of the thought'.
- II. A thought is (a) objective, (b) the sense of an indicative sentence. . . .
- III. A thought must have at least one 'unsaturated' or functional element, otherwise its elements would fail to coalesce and would remain merely disparate atoms.
- IV. In a thought the complete elements refer (if at all) to objects.

The nature of this enterprise makes it hard to evaluate its success. There are many issues one would think to be central to any attempt to offer a theory of judgement which Bell nevertheless does not address. For instance, although he argues that it would be wrong to require an account of judgement to restrict itself to the form of an account of the propositional attitude constructions used to attribute judgements to others, he does not justify the book's failure to present any such account as a proper part of such a theory. Again, although it has been suggested above that Bell was not obliged to restrict his attention to the notion of assertoric force (the analysis of the act of judging), one would certainly like a fuller and more satisfactory account of that notion than the cursory sketch we are offered.<sup>14</sup> The book does its work in a sort of methodological no-man's land between textual exegesis and theory construction owing allegiance only to the phenomena it seeks to theorize about.

This is not to say that the analysis is not enlightening, however. Bell is at his best when dissecting the explanatory role assigned by Frege to his technical concepts. When he succeeds we learn both about Frege and about the phenomena. Consider for instance the notion of *Bedeutung*. Bell tells us that:<sup>15</sup>

. . . Frege had not one, but two notions of reference. These notions hang together so well in the case of singular terms that they are hard to distinguish in this context. In the case of predicates, however, they are not only distinguishable, they are difficult to reconcile. One notion is this: the reference of an expression is that extra-linguistic entity with which the expression has been correlated or which it picks out. The other notion of reference is that it is a property which an expression must possess if that expression is to be *truth-valuable* (to coin a phrase). By truth-valuable I mean such that it either possesses a truth-value, or is capable of being used (and not just mentioned) in a sentence which possesses a truth-value.

Bell claims that although in the case of singular terms one notion can play both of these roles, since for them to be truth-valuable just *is* to be correlated with an object, in the case of sentence and functions the two notions diverge. All that Frege ever offers in the way of evidence for the application of the notion of reference to expressions in these categories is considerations showing them to be truth-valuable. Since he does not distinguish the two different notions of reference which he has in play, he feels entitled to conclude that they possess reference in the first sense as well. But this is a *non sequitur*, or at any rate a transition which must be justified, and not simply assumed on the basis of the conflation of the two different senses of *Bedeutung*. Thus Bell rejects the notion of

truth-values as objects, and of functions as the references of functional expressions, as excess conceptual baggage mistakenly mixed in with the second notion of reference, which is the only one doing any explanatory work for these categories.

This analysis is clear-headed and valuable but can be faulted on two grounds, each of which amounts to a request for further analysis. First, as Dummett points out,<sup>16</sup> the characterization of the second notion of reference does not seem right. For as Bell has described it, reference is a property which an expression either has or lacks, depending upon whether sentences containing it can have or always lack truth values. But Frege's notion is that in addition to having or lacking reference, expressions which have reference can have *different* references, accordingly as they make different contributions to the truth-values of sentences containing them. The test is always substitutional—two expressions which have reference have different references if and only if in some context the substitution of one for the other changes a true sentence into one which is not true. Others who have noticed the distinction Bell is after have put things better. For instance, Tugendhat<sup>17</sup> (who seems to have introduced this line of thought) calls this non-relational sense of reference “truth-value potential” and in effect identifies the truth-value potential of a sub-sentential expression with the equivalence class of expressions intersubstitutable *salva veritate*.

The sharpening of Bell's distinction (which makes it similar to that between ‘referent’ and ‘reference’ which Dummett uses throughout [8]) does not affect his criticism of the inference from possession of reference in this non-relational sense to possession of reference in the relational sense, of course. But it does affect a further use he wants to make of the distinction, to argue that it is incorrect to think of predicate

expressions as having a reference at all, even in the non-relational sense. For here Bell argues that Frege incorrectly takes as a necessary and sufficient condition for the truth-valuability (in Bell's sense) of predicates that they have sharp boundaries. He accordingly takes it that the assignment of reference to predicates is motivated only by this requirement, and so that showing the untenability of such a requirement is sufficient to show the inappropriateness of assigning reference to predicate expression at all. This line of argument is undercut by seeing that there is more to the second notion of reference than truth-valuability. Since the denial of the cogency of the application of the notion of reference to predicates (or function expressions generally) is one of the main innovations of Bell's analysis, his failure adequately to characterize that part of Frege's notion of reference which remains when one takes away correlation with an extra-linguistic object has serious consequences for the subsequent course of his argument.

Dummett, however, rejects not only Bell's characterization of the second notion of reference, but also the claim that there are *two* notions of reference. He claims that the relational and the nonrelational senses represent "two ingredients of *one* notion". The second "tells us what Frege wanted the notion of reference *for*, and the other tells us how he thought that it applied to the various categories of expression".<sup>18</sup> It may be granted that the explanatory work Frege wanted the notion of reference for is its truth-value potential or contribution to the truth-conditions of sentences, and that he thought that the intersubstitutability equivalence class of equipollent expressions was determined by the correlation of all and only its members with the *same* extra-linguistic entity. But it would still remain to be asked, for instance, whether the identity of the correlated object and the nature of the correlation can be inferred from the semantic equivalence class of

expressions they determine, as Frege's arguments concerning the reference of sentences and functional expressions would seem to require. Such a question is in no way made less urgent or easier to answer by rephrasing it in terms of two ingredients of one notion rather than in terms of the relations of two notions. In the final section of this paper it will be argued that this difficulty is one instance of a quite general definitional failure of Frege's part, one which in another context he tried unsuccessfully to resolve in a purely technical way.

Putting the issue in these terms raises the second source of dissatisfaction with Bell's argument. For the sort of question just raised seems no less important or difficult for the paradigmatic case of singular terms than for the parts of speech Bell finds problematic. The basic substitutional/inferential methodology which yields the non-relational sense of reference as an equivalence class of expressions vastly underdetermines the correlated objects and mode of correlation invoked by the relational sense even for proper names. Tugendhat, having formulated the nonrelational notion of reference, takes it to be *the* notion of reference, discarding correlation with an object as a realistic confusion best extruded from Frege's thought. Sluga follows Tugendhat in this regard. The reason in each case is that all that Frege's analysis of the use of expressions seems to require is the sorting of expressions according to the non-relational sense of substitutional role. The semantic analysis he developed is a method for the perspicuous codification of inferences. Truth is what is preserved by good inferences, and subsentential expressions can be grouped into co-reference classes accordingly as intersubstitution within the classes preserves such good-inference potentials. Such an approach can give rise to specification of the conditions under which two expressions

have the *same* reference, but how can it warrant a claim that the shared reference is to be identified with some object (among all those which in one way or another could be taken to determine the same coreference classes) specified otherwise than as the reference of an expression? The answer seems to be that Frege's arguments for this identification are straightforwardly substitutional ones, in particular that for any singular term *t* we can always substitute (saving the inferential potentials) the term *the object referred to by the singular term 't'*. The expressions which license intersubstitution of expressions are identity locutions (as Frege had argued in the *Grundlagen*), and so we are correct to say that the object referred to by the singular term 'Julius Caesar' *is* Julius Caesar. Whether this fact has the significance Frege thought it had is another matter.<sup>19</sup>

One of the most important discoveries of the early 1970's, both from the point of view of the interpretation of Frege and of the philosophy of language generally (for once, made independently of Dummett) concerns the need to distinguish two different explanatory roles which are conflated in Frege's technical concepts of sense. Kripke and Putnam independently argued<sup>20</sup> that the *cognitive* notion of the sense of an expression, what one who has mastered the use of that expression may thereby be taken to understand and the *semantic* notion of the sense of that expression, what determines the reference of the expression, cannot in general be taken to coincide. In particular, in the case of proper names no knowledge or practical capacity which can plausibly be attributed to an ordinary competent user of the name will suffice to determine the object of which it is a proper name. A similar point can be made about the use of natural kind sortals. Since Frege had required that his notion or the sense of an expression play both the cognitive and the semantic role, and since for an essential range of expressions no single notion can

do so, it is apparent that his concept must be refined by dividing it into two distinct sense-concepts, whose interrelations it then becomes urgent to investigate.

A further distinction within the semantic notion of sense has been urged by a number of writers, on the basis of the consideration of the behavior of indexical or token-reflexive expression.<sup>21</sup> In Kaplan's idiom, we must distinguish for such expressions between their *character*, which is associated with the expression type, and the *content* associated with each contextually situated token(ing) of that type. The distinction in question is evident in the following dialogue:

A: I am anxious to get started.

B: No, it is possible that you are eager, but I am the anxious one.

We are concerned with the semantic notion of the sense of an expression, that is, with the way in which its reference is determined. In one sense both tokens of "I" have their reference determined in the same way, for in each case it is the speaker responsible for the tokening who is referred to. These expressions share a character. But in another sense A's token of "I" and B's token of "you" have their reference determined in the different ways ( e.g. for the purpose of tracking the referent through the other possible worlds which must be considered to evaluate the modal qualifications in B's remark). The referents of these tokenings will coincide in every possible world relevant to the evaluation of these utterances, in virtue of the identity of their contents. The characters of these expressions, together with the context in which they are uttered, determine a content which in turn determine a referent in every possible world. It is this latter task with which the semantic notion of sense is charged for nonindexical expressions. Such expressions may accordingly be thought of as those whose character determines a content

without needing to be supplemented by a context. The point is that as we ask about what would be true in other worlds of the individual picked out by E's indexical utterance there is a *double* relativity to possible worlds, accordingly as those worlds can be relevant to the two different stages in the determination of a referent. First, since B's remark could have been addressed to someone other than A, we must consult the world-context in order to determine what content is fixed by the character of the expression when uttered in that context. The individual concept so determined as a content can then be tracked through various possible worlds and assigned referents in each, so that modal claims can be evaluated.

Without referring to either of these antecedents, Bell distinguishes two notions of expression sense in a way which partakes of some of the feature of each of the other distinctions. He calls his two notions "input sense" and "output sense", and introduces them by reference to two Fregean principles:<sup>22</sup>

PS1: The sense of a sentence is determined by the senses of its component parts,  
and

PR1: The truth-value of a sentence is determined by its sense. (And, of course,  
how things stand.)

His claim is that although the "two principles depend for their plausibility and usefulness on there being a sense of 'sense' which remains constant throughout", in fact they demand different ones. Input sense is that notion of which principle PS1 holds, and output sense is that notion of sense of which PR1 holds. Input sense is that which is preserved by correct translations and that for which synonymy claims assert identities of sense. Sub-sentential expressions have input senses ("meanings"), and these combine to

determine the input senses of sentences containing them. Output senses are defined as what is common to claims such as “Today I ate plum pudding,” and “Yesterday you ate plum pudding”. The input senses of sentences together with a context of utterance determine such output senses. The output senses of sentences are what can meaningfully be described as true or false, as per principle PR2.

As described so far, Bell’s distinction amounts to the claim that the cognitive/semantic and character/content partitions of the notion of sense ought to be seen as coinciding. For the compositionality of sense’ is a postulate required for the explanation of the possibility of understanding complex expressions, so that it must be input senses which are: in the first instance grasped cognitively. Semantic senses, determining truth values of sentences, are in turn identified with output senses. But since the latter are determined by the former together with a context of utterance and the distinction is enforced by attention to indexical expressions, the character/content distinction is likewise subsumed by the difference between input and output senses.

Such an identification is clearly subject to a number of objections, as consideration of the quite different motives and functions of the conflated distinctions indicates. But these difficulties may not be insurmountable. Perhaps a useful view could be elaborated based on the assimilation of the sense in which the referent of a proper name token is determined not by what its utterer understands by it, but only by this together with a causal, historical, and social context in which the token is embedded, on the one hand, and the sense in which the reference determining sense of a token of “yesterday” is given not just by what one can understand as the meaning associated with the expression type, but only this together with a concrete context of use. But Bell does

not attempt to develop such an account. In part this is because he has nothing whatever to say about what “contexts” are, or how these together with input senses determine output senses. And it is just here that all the detailed work is involved in making out either half of such an assimilation, and hence in justifying their conflation. But Bell is precluded from addressing such a task by other, less defensible features of his view.

For Bell denies that sub-sentential expressions have output senses at all, claiming that “output sense is essentially sentential”.<sup>23</sup> No argument or even motivation for this position is presented. It is suggested that for sentences the distinction between input senses and output senses corresponds to that between sentences and the statements they can be used to make, and that it is better to think of the former not as possessing truth-values which change, by contrast to statements whose truth-values do not, but rather to think of the former as not the kind of thing which can have truth values at all. But no reason is given for not extending this distinction to sub-sentential expressions. The distinction between the two varieties of sense is introduced, as indicated above, in terms of two Fregean principles. PR2, the ‘sense determines reference’ principle, is quoted at this portion of the argument as restricted to sentences and truth-values. But of course the principle Frege uses is not so restricted. Indeed, when Bell first introduces it some sixty pages earlier it is in unrestricted form. He has just been discussing the principle he calls PR1, that the reference of complex expressions is determined by the references of their components (which Bell discards because as we have seen he does not attribute reference of any kind to functions). He says:<sup>24</sup>

Elsewhere in his writings, however, he seems to invoke a quite different principle which we can call PR2. It is this: (a) the reference of any

expression is determined by its sense, (b) the sense of a complex expression is determined by the senses of its component parts.

Two features of *this* definition deserve comment. First, part (b) of principle PR2 as here stated is what he later calls PS 1 and is concerned precisely to distinguish from PR2. Second, part (a) of *this* original statement differs from the later version in not being restricted to sentences. Neither of these substantial changes in the significance of his expression “PR2” is announced, acknowledged, or motivated in the intervening text. Such carelessness in specifying a central interpretive principle which one has taken the trouble to name for clarity of reference is bad enough under any circumstances. It is unforgivable when essential features of one’s own claims and their justifications depend precisely on the matters obscured by the sloppiness. As things stand, the reader is left with no idea why in using the two principles PR2 and PS1 (= PR2(b) in earlier statement) to distinguish two notions of sense one should employ the later version of PR2 rather than PR2(a) from the earlier version, which is the principle Frege endorsed. Apart from the invocation of PR2, output senses are specified as what is common to the two “plum-pudding” sentences quoted above. As our sketch of the character/content distinction show, it is not at all obvious why this characterization should not extend to what is common to ‘today’ and ‘yesterday’, on the one hand, and ‘I’ and ‘you’ on the other.

Bell does, however, employ the restriction of output senses to sentences to argue for a further point. For he claims that the “context principle” of the *Grundlagen* may be understood in terms of the fact that terms only have input senses, which together with the input senses of other expressions determine sentential input senses, which in context determine a truth-value. Since the reference of terms matters only in determining truth-values, it is “only in the context of a sentence that a term have a reference”. Clearly

nothing can be made of this line of thought in the absence of a rationale for its basic premises.

These difficulties with the distinction between input senses and output senses also make it difficult to evaluate another novel interpretive suggestion which Bell offers. He concludes his discussion of the senses of proper names with the claim<sup>25</sup> “The sense of a proper name, then, is that it purports to refer to a determinate object of a given sort with which it has been conventionally correlated.” The sense of a proper name is here taken as “that which one understands when one is able to use it correctly”.<sup>26</sup> As indicated above in the discussion of the relation of the cognitive notion of sense to Bell’s notions, this must be the input sense, for subsentential expressions aren’t supposed to have output senses. It is accordingly obscure what the connection is supposed to be between the senses Bell is offering a theory of here and the determination of referents for the proper names they are senses of. What then are the criteria of adequacy for an account of what a name user must be taken to understand? Bell examines the conditions under which we would want to deny that someone had mastered the use of a name, and concludes that in addition to using it as a singular term one must at least know some sortal under which the referent is taken to fall in order to be judged a competent user. This is useful as a necessary condition, but much less plausible as a sufficient condition to be taken to be using an expression as a proper name. For a sufficient condition would seem to require that one be appropriately connected to a community of users of the name, perhaps an historically extended one, whose joint use *does* determine a referent, though no individual’s use need do so. It is not obvious that merely believing that some conventional correlation has been established with an object of the right sort is sufficient

to be appropriately connected with the community of users of that name. In any case, to argue for such a principle would require looking at how input senses and various specific sorts of context can together determine output senses and eventually referents for the names in question, and this Bell does not undertake.

Bell wants his notion of proper name sense in order to develop an appropriate account of the senses of functional expressions. This latter task is made especially urgent by the confrontation between his denial that the referents of functions have any explanatory value, on the one hand, with the undeniable importance in Frege's scheme of functions and concepts understood as functions, on the other. Bell's reconstruction reconciles these ideas by interpreting concepts and functions as the *senses* rather than the references of functional expressions. A concept, accordingly, is to be understood as a function which can take as arguments proper name senses of the sort he has described, and yield thoughts, the senses of sentences. While this identification of concepts must be seen as a revision rather than an interpretation of Frege's thought, it might seem that, setting that identification aside, at least the account of the senses of functional expressions as functions from the senses of argument expressions to the senses of value expressions ought to be uncontroversial. It is not, and it is instructive to see why not.

As Bell has pointed out in his discussion of senses generally, the concept of sense is required to play two distinguishable roles. First, the sense of a component of a complex expression must contribute to the determination of the sense of that complex. But also, the sense of the component must determine a reference for that component. This gives us two different ways to think about the senses of functional expressions such as predicates. On the one hand they must combine with the senses of terms to yield the

senses of sentences. On the other hand they must be the way in which a function from objects to truth values is determined or given. It is not obvious that these two jobs can be done by one notion. In particular, Dummett has argued<sup>27</sup> that “once the proper name has specified the way in which the object is given, then it has made its contribution to the sense of the sentence; if it had not, then it would be impossible to see how its sense could *both* contribute to the sense of the sentence *and* consist in the way in which the object is given.” That is, maintaining the coincidence of the two roles of sense in the case of proper names (presumably where our grasp is firmest) commits us not only to their divergence for functional expressions, but also to which half we give up, namely the identification of their senses with sense functions. Geach has objected to this doctrine of Dummett’s,<sup>28</sup> and it is instructive to examine Dummett’s response.<sup>29</sup>

It is not disputed that once a sense has been assigned to a predicate, a function from the senses of proper names to thoughts is determined. For according to Dummett the predicate sense is the way in which a function from objects to truth values is given. Hence, when that function is supplemented by an object, it determines a way in which a truth-value is given, that is, a thought. But since a term sense will determine such a supplementing object (according to the second role of senses mentioned above), the predicate sense will induce indirectly a function from term senses to sentences senses. As Dummett says, “the question is whether the sense of the predicate just *is* that function.”

To argue that it is not, Dummett appeals to a further thesis of Frege’s about senses, namely that the senses of component expressions are *parts* of the senses of the complex expressions in which they occur. We have seen that it is a mistake to think of

functions or their arguments as parts of the values they generate, as Frege's retraction of his careless claim that objects are parts of truth values shows. But since Frege did hold that predicate senses are parts of thoughts, we would be committing precisely this howler if we identified those senses with functions taking term senses into thoughts. This is an ingenious counter-argument, but it cannot be considered decisive. For while it would be a howler to treat functions and their arguments generally as parts of the values they determine (as in the combination of Sweden and the function *the capital of . . .* to yield Stockholm), this consideration does not show that particular functions and kinds of function cannot have values which contain the functions or their arguments as parts. Stockholm is part of the value of the function *the country of which . . . is the capital*. And mathematical examples of function-values which contain functions as parts in the set-theoretic sense are easy to come by. (One thinks of the story of the oracle who offered to answer a single question, and upon being asked "What is the ordered pair whose first element is the best question I could ask you, and whose second element is its answer?" replied (falsely, I suppose): "the ordered pair whose first element is your question and whose second element is this answer .")

Insulated from this dispute about sense functions by his distinction between input senses and output senses, Bell backs up his commitment to treating the senses of functional expressions as functions by citing a number of passages, both published and from the posthumous works, in which Frege unequivocally describes such senses. as "unsaturated", "incomplete", and "in need of supplementation", going so far in fact as to say that "The words 'unsaturated' and 'predicative' seem more suited to the sense than to the reference".<sup>30</sup> To motivate his identification of concepts with sense functions, Bell

argues as follows.<sup>31</sup> The only reason Frege had for believing in concepts as predicate referents was the need to deal with a situation in which predicates have a sense and so determine a thought, but lack a reference, and so determine a thought which has no truth-value. The only case where this can happen which does not reduce to the failure of a term to have a reference is where the predicate is not defined for the sort of argument to which it is applied. But this sort of case can be much more plausibly excluded by considerations concerning predicate *senses*. For such cross-categorial predications (such as “Julius Casesar *is* the sum of two prime numbers”) ought properly to be seen as not succeeding in expressing thoughts at all. Bell’s solution accordingly is to see predicates as having sortal restrictions associated with their argument places, which together with the ‘sortal physiognomy’ he has already assigned to proper name senses yields the result he desires. One of the benefits which might be derived from such a radical reconstruction should be made manifest by the discussion to be given below to the difficulties ensuing from Frege’s insistence that functions be defined for all arguments whatsoever. However, as before, the evaluation of this thesis about senses must await some resolution of the general questions Bell has left open concerning his distinction between input and output senses.

## II

Hans Sluga’s new book on Frege in the “Arguments of the Philosophers” series<sup>32</sup> represents an approach complementary to Bell’s in almost every regard. Its central aim is to reread Frege’s work in the light of that of his precursors and contemporaries, rather than by reference to his successors in the analytic tradition, as has been traditional. Although Frege’s unprecedented innovations in symbolic logic have made it natural to

think of him exclusively in the role of the founder of a tradition—as a man without a past—Sluga argues that we ignore at our peril his intellectual climate and the influences which conditioned various aspects of his technical concepts and of the explanatory tasks he set for them. Sluga's task is not purely historical, however. For he is also concerned to set out and justify novel readings of some of Frege's purely philosophical doctrines, readings which are suggested and motivated by the historical recontextualization he recommends. The result is a stimulating new picture of Frege's thought which will be of interest even to those who are not in the end persuaded in detail by it. Furthermore, since the narrative strategy employed is to trace the development of Frege's ideas chronologically (starting, as it were, before he was born) and surveying all of his important writings *seriatim*, this book is excellently constructed to serve as an introduction to these ideas (as Bell's or Dummett's books, for instance, could not) as well as to challenge specialists.

The book's historical orientation, then, is not adopted only for its own sake, but also in order to guard against blinding ourselves to interpretively significant features of Frege's work by the importation of anachronistic prejudices. Accordingly, it is primarily in terms of the philosophical illumination they provide for our appreciation of Frege's concepts and claims that we must evaluate the success of Sluga's various invocations of historical influence. The claimed influences may be considered under four headings. First, a view is presented about who Frege took to be his philosophical opponents. Next, Leibniz is identified as a precursor. Third, claims are made about the influence of two logicians of the generation preceding Frege's, Lotze and Trendelenburg. Finally and most significantly, it is claimed that overlooking the intellectual debt which Frege owes

to Kant has most seriously distorted our understanding. We will consider these claims in this order.

In his first chapter, Sluga is concerned to refute the claim that “In a history of philosophy Frege would have to be classified as a member of the realist revolt against Hegelian idealism, a revolt which occurred some three decades earlier in Germany than in Britain.”<sup>33</sup> In this aim he succeeds unequivocally. Hegelianism had ceased to be dominant or even popular in German philosophical circles some years before Frege was born. The view against which Frege was reacting is the scientific naturalism which Sluga claims was held by the physiologists turned philosophers Vogt, Moleschott, Buchner, and Czolbe, popularized during Frege’s lifetime by Haeckel, and shared with some reservations by Gruppe. Ontologically this view is a reductive materialism, and epistemologically it is an empiricist psychologism. Sensations are viewed as material processes of the brain. Concepts, and hence the thoughts constructed from them are taken to be reflections of such sensations. Logic is seen as the study of the laws of thought, that is, as an empirical investigation seeking to establish the natural laws governing the association of concepts in judgment and of judgments in inference. It is this psychologism which Frege so vigorously opposed, and on those relatively few occasions when he describes his opponents as ‘idealists’ it is clearly this school which he has in mind.

This is a point of no small moment, especially in the context of an evaluation of Frege’s role as progenitor of the analytic tradition. For his over-arching objection to the naturalists is their failure appropriately to distinguish between the normative and ideal order of correct inference and justification on the one hand, and the descriptive and actual

order of causation and empirical processes on the other. Their concomitant confusion of features of cognitive acts with features of the contents of those acts is merely the expression of this original sin. And in his insistence on the centrality of this basic distinction Frege is at one with Kant and the post-Kantian idealists, and at odds with the primarily physicalist and empiricist tradition in Anglo-American philosophy which he fathered, and in the context of which it has been natural for us to read him.<sup>34</sup>

Throughout his book Sluga talks about Leibniz' influence on Frege, but when he specifies the details of this influence his claims turn out to be quite weak. Like Leibniz (and Kant), "Frege is interested in the study of logic and the foundations of mathematics because they allow one to ask in a precise form what can be known through reason alone."<sup>35</sup> Aside from this general rationalist commitment to the possibility of *a priori* formal knowledge, the only Leibnizian doctrine which is attributed to Frege is the endorsement of the project of the universal characteristic. Frege explicitly describes the motivation for his *Begriffsschrift* in this way. That at this level of generality Frege owes a debt to Leibniz is hardly a novel or surprising claim, however. Sluga also discusses the influence of Trendelenburg, but in the end the claims seem to come to little more than that he was the conduit through which Frege became familiar with Leibniz' ideas.

It is otherwise with the connection discerned between Frege and the logician Lotze. The suggestion of influence here has specifically been denied as "a remarkable piece of misapplied history".<sup>36</sup> Yet in this case Sluga shows sufficiently striking similarities to make the hypothesis of influence persuasive. It is known that Frege read Lotze. Indeed it has been argued that the theory of judgement in opposition to which he presents his innovation in the *Begriffsschrift* just is Lotze's formulation.<sup>37</sup> The essay

immediately preceding “The Thought” in the journal in which it was originally published, which Sluga takes to have been intended by the editors as an introduction to Frege’s essay, mentions Frege in the context of an exposition of Lotze which highlights several Fregean doctrines.<sup>38</sup> From Sluga’s account of Lotze’s views (as presented in the *Logik* of 1874 and an earlier work of 1843) one can extract eight points of similarity with Frege.

First, Lotze inveighs against psychologism and indeed is the figure Frege’s contemporaries would probably have identified as leading the battle against the dominant naturalism of the day and in favor of a more Kantian position. Next Lotze was a logicist about mathematics, although there is no hint in his works that he took the detailed working out of such a reduction to logic as part of what would be required to justify this view. Third, Lotze insists, against empiricistic sensationalism, upon the distinction between the objects of our knowledge and our recognition of such objects, in much the same terms that Frege did. Fourth, Lotze emphasized and developed the Kantian strategy of explaining concepts as functions (though of course he does not have the notion of functions as unsaturated which Frege derived from his own substitutional method of assigning contents to sub-sentential expressions). Fifth, Lotze attacks the empiricists with a distinction between the causal conditions of the acquisition of concepts and the capacity to use such concepts in correct reasoning which mastery of the concepts consists in (see note 34 above). Next, Lotze offers a theory of identity statements according to which the two terms share a content, but differ in form. This is the *Begriffsschrift* view, and the language survives into the opening paragraphs of “Über Sinn und Bedeutung.” Seventh, Lotze endorses the Kantian principle of the priority in the order of explanation of judgements to concepts which Frege endorses in the *Grundlagen*. Lotze does not

succeed in being entirely consistent on this point, since he also is committed to atomistic principles which are not obviously compatible with the view on the priority of judgements. Although Sluga does not say so, those who take Frege not to have discarded the context principle in the post-1890 writings must find a similar tension in some of the procedures of the *Grundgesetze*. Finally, Lotze is committed to the objectivity of sentential contents, and treats them as neither mental nor physical just as Frege does. Lotze, however, specifically denies that this objectivity is grounded in the correlation of sentences with objects such as Frege's thoughts appear to be, taking a more Kantian position. Sluga, as we shall see below, argues that despite apparent statements to the contrary we should understand this to be Frege's view as well.

This is a suggestive set of similarities to find in a prominent near-contemporary logician with whose work Frege was familiar. Recognizing them as important need not commit one to minimizing the significant, perhaps dominant, differences in outlook which remain between Lotze's revived Kantianism and Frege's philosophical elaboration of his semantic methodology (although Sluga does on occasion succumb to the temptation to treat Frege's agreement with Lotze on one point as evidence that he probably agreed with him on others). Only according to the crudest notion of what philosophical originality consists in is there any incompatibility between finding enlightenment in the demonstration that these general principles were in the air and so came complete with a history and a tradition, on the one hand, and the appreciation of the genius shown in the use such adopted and adapted raw materials were put to in service of quite a different explanatory project on the other.

Sluga's most important and sustained argument, however, concerns the influence of Kant on Frege. He claims that Frege should not be thought of as a dogmatic realist about physical objects nor as a Platonist about abstract objects, as he almost universally has been thought of. He should be seen rather as a Kantian whose realistic remarks are to be interpreted as expressing that merely empirical realism which is one feature of transcendental idealism. This is certainly a radical reinterpretation. What evidence can be adduced for it? Sluga's considerations may be assembled as five distinct arguments.

First it is pointed out that Frege joined a philosophical society whose manifesto is explicitly idealist and Kantian, and that he published in their journal. By itself, this shows little, for Frege had so much trouble getting his work into print and finding others willing to discuss it that we cannot be sure how much he would have put up with to secure such opportunities. The rationale Sluga suggests<sup>39</sup> is that "what tied him to the idealists was primarily his opposition to the various forms of naturalism". Specifically, Frege and the idealists (a) were anti-psychologistic, (b) endorsed an objectivist epistemology (taking the contents of judgements to be independent of their entertainment by thinkers), and (c) endorsed a rationalistic a priorism about mathematics. These points are well taken, but the views involved are all consistent with Platonism and realism generally as well as with transcendental idealism. Indeed Sluga admits that "one can read much of Frege and not raise the question of transcendentalism". So we must look elsewhere for a warrant for such an attribution.

The second argument concerns Frege's attitude towards the truths of geometry.<sup>40</sup> It is remarked to begin with that in his *Habilitationsschrift* Frege held a Kantian view on this topic, saying that geometry rests "on axioms that derive their validity from the nature

of our capacity for intuition (*Anschauungsvermögen*)". Furthermore, throughout his career Frege describes geometrical knowledge as synthetic a priori, and on this basis rejects non-Euclidean geometry as *false*. From this fact Sluga concludes: "Frege held a Kantian view of space and hence a transcendently subjective view of the objects that occupy it." The only elucidation offered of this crucial 'hence' is the later statement that "Frege's view must be close to Kant's: Empirical objects are in space and time, but space and time are a priori forms of sensibility. That seems to imply that for Frege empirical objects can only be empirically real, but must be transcendently ideal." That Kant believed the two views to be linked in this way falls far short of showing that Frege did so. Certainly such an argument cannot be taken to undermine an interpretation which takes Frege's realistic remarks about physical objects at face value, and admits that his views are inconsistent to the extent that he never confronted these latter with his views about geometry with an eye to reconciling them. On the other hand some interpretive cost is clearly associated with attributing such an inconsistency to Frege.

The next two arguments must be judged less satisfactory.<sup>41</sup> First, Sluga argues that in the context of Kantian transcendentalism (as just discussed) Platonic realism looks like dogmatic metaphysics. So Frege should have been expected to argue that views (a) through (c) above, on which he argues with the idealists, cannot in fact be warranted transcendently. But Frege nowhere argues this. The trouble with this argument is that there is no evidence that Frege did not, as most of his contemporaries did, read Kant's transcendentalism as a form of psychologism. If he had done so, he would have dismissed it and so not felt the force of the demand in question. Sluga next argues that every claim of Frege's that can be taken as evidence of Frege's realism can be matched

by a passage in Lotze, who had a Kantian idealistic theory of validity. This argument seems to do no more than restate the point that (a) through (c) are consistent with either position. For it is a criterion of adequacy of anyone's transcendentially idealistic position that it have room for all of the claims the realist wants to make, suitably reinterpreted. Further, Frege does insist that thoughts are independent, not just of this thinker or that, but of the very existence or even possibility of thinkers at all. This seems to contradict Lotze's account of objectivity as rule-governed intersubjectivity.

Sluga's final argument is weightier and involves more interpretive work, both in construction and evaluation. The basic claim is that "there are strewn through Frege's writings statements that appear irreconcilable with Platonic realism. In particular the central role of the Fregean belief in the primacy of judgements over concepts would seem to be explicable only in the context of a Kantian point of view."<sup>42</sup> Arguing in this way obviously commits Sluga to showing that Frege does not discard the context principle when he arrives at the distinction between sense and reference. We will see below that he contributes significant new considerations to that debate in furtherance of this aim. But the incompatibility of realism with the recognition of the primacy of judgements must also be shown. The latter view is 'Kantian', but it does not obviously entail transcendental idealism, which is the view in question. Sluga takes the principle of the primacy of judgements to serve the purpose for Kant<sup>43</sup> of refuting any atomistic attempt to construct concepts and judgements out of simple components, and in particular to resist the empiricist sensationalist atomism of Hume. Such a view is indeed incompatible with the reism of Kotarbinski (to which Tarski's recursive semantics owes so much), which sees the world as an arrangement of objects out of which concepts and judgements

must be constructed set-theoretically.<sup>44</sup> But the Kantian principle need not be taken to be incompatible with Platonic realism about abstract entities such as *thoughts* which are the contents of judgements. Given that the context principle does not show that Frege was a transcendental idealist about thoughts, it seems also open to him to hold some form of realism about other objects, provided thoughts retain an appropriate primacy (as, given the very special status of truth in the late works, even those who see the context principle as discarded are committed to granting) even if he has not discarded that principle. So if the case for the persistence of the context principle can be made out, it should be taken as showing that Frege was a Kantian in the sense of holding the context principle, not in the sense of being a transcendental idealist.

Still, this point is worth establishing for its own sake. Sluga correctly sees the *Begriffsschrift* as the confluence of three lines of thought: (1) that judgements, as involved in inference, are the original bearers of semantic significance, so that it is only by analyzing such judgements according to the procedure of “noting invariance under substitution” that such significance can be attributed to sub-sentential expressions (‘the primacy of judgements’), (2) the Leibnizian idea of a perfect language, and (3) the idea of reducing mathematics of logic. Assuming the context principle was thus “anchored deeply in Frege’s thought, it is implausible to conclude with Dummett that in his later years Frege simply let it slip from his mind.”<sup>45</sup> Sluga advances five arguments for the persistence of the principle, and along the way addresses two commitments of Frege that have been taken to be incompatible with such persistence.

First, Sluga offers an important consideration which has not previously been put forward in the extensive literature discussing this question. The first of the 1891-2 essays

that Frege wrote is a seldom read review of L. Lange's *The Historical Development of the Concept of Motion and its Foreseeable End Result* called 'The Principle of Inertia'. In it Frege argues at some length that the concepts of a theory are not given prior to and independent of that theory. Rather those concepts can be arrived at only by analyzing the contents which the judgements constituting the theory are given by the inferences concerning them which that theory endorses. This is a significant new piece of evidence supporting Sluga's view. The only question which might be raised about it is that since this semi-popular piece does not deploy the full-blown apparatus of sense and reference it may be wondered whether the views there expressed were confronted by Frege with that apparatus, or whether the essay might not be seen as merely the latest of his early works. But to take such a line would be to concede a lot, and future claims that the context principle was discarded will have to confront this argument of Sluga's in detail.

Next Sluga offers a novel reading of the essay on the distinction between sense and reference which denies that, as has often been claimed, that distinction as there presented applies primarily to singular terms and their relations to the objects which are their referents, and hence commits Frege to an assimilation of sentences to terms which is incompatible with the context principle. The strategy here is, in effect, to deny that '*Bedeutung*' as Frege uses it *ever* has the relational sense which indicates correlation with an object. Relying on the Tugendhat essay mentioned above in connection with Bell, Sluga understands '*Bedeutung*' as a non relational semantic potential defined paradigmatically for sentences, in virtue of their role in inference. The introduction of this notion in the context of the consideration of identities involving singular terms is seen as a rhetorical device of presentational significance only. In the final theory

subsential expressions are taken to inherit indirect, inferential significances in virtue of their substitutional behavior in sentences, which alone are directly inferentially and hence semantically significant. Thus *'Bedeutung'* is paradigmatically a sentential notion.

To this analysis is conjoined an account of *'Sinn'* as a cognitive notion, as what matters for knowledge. But again, the units of knowledge are judgements, and subsential expressions can become relevant only insofar as they can be put together to form sentences which can express judgements. So sense also should be seen as primarily a sentential notion, which applies to subsential expressions only in a derivative way. This line of thought concerning senses is then combined with that concerning reference in a subtle and sensitive account of the puzzling relations between the Lotzean rendering of identity locutions offered in the *Begriffsschrift* and its successor in *'Über Sinn und Bedeutung'*.

The previous discussion of Bell's interpretation suggests that these readings leave something to be desired. Sluga does not acknowledge the existence of any passage or considerations indicating that Frege does have a relational notion of reference in play. Yet such passages and considerations do exist, and merely elaborating the nonrelational version of Frege's concept, as Sluga does, does not obviate the necessity of investigating the relations between the two notions and the possibilities for reconciling them. Similarly, Sluga pushes his discussion of the notion of sense no farther than the discrimination of the cognitive role played by that concept. He has nothing to say about the semantic notion of sense, or accordingly about how senses are to be understood as determining references, even nonrelational references. On these points Sluga's analytic net does not have as fine a mesh as Bell's. As a result, his ingenious interpretation of

sense and reference will require further filling-in before its eventual promise can be assessed.

The overall interpretation which results from all of these arguments, however, is challenging and powerful. The primary objections to the persistence of the context principle are that Frege nowhere explicitly endorses that principle after the 1884 *Grundlagen* formulation, and that the principle is incompatible with two central doctrines of the 1891-92 essays, namely the semantic assimilation of sentences to terms and the account of concepts as functions from objects to truth-values. Sluga claims that his readings of the “Inertia” essay and of USB meet these objections. He does not say in detail how the doctrine about functions is to be reconciled with the context principle, but does argue that the “Inertia” essay justifies us in attributing *Bedeutung* to any expression which makes an appropriate contribution to the possession of truth values by sentences containing it. Thus function-expressions may be assigned (nonrelational) reference on this account. Using intersubstitution equivalence classes to move from Tugendhat’s nonrelational sentential semantic significances to those of sub-sentential expressions does indeed justify such an attribution. But in the “Inertia” essay, Frege seems to be using “concept” in the ordinary sense rather than his technical one, that is, to refer to the *senses* of predicate expressions rather than their references. This being the case, it is not clear how the envisaged reconciliation of the context principle with the view of concepts as functions from objects to truth values is to be achieved.

Besides the evidence of the essay on inertia, Sluga offers two further reasons to deny that the later Frege is silent on the topic of the context principle. First, he mentions in several places the posthumously published ‘Notes for Ludwig Darmstaedter’ (of 1919)

as showing that Frege continued to endorse the principle. He does not say what passages he has in mind, but he presumably intends the following:<sup>46</sup> “What is distinctive about my conception of logic is that I begin by giving pride of place to the content of the word ‘true’, and then immediately go on to introduce a thought as that to which the question ‘Is it true?’ is in principle applicable. So I do not begin with concepts and put them together to form a thought or judgement; I come by the parts of a thought by analyzing the thought.” Such a passage does show that sentences play a special explanatory role for the late Frege, but that much is not in question. At most such claims would show that a version of the context principle held for *senses*, confirming Sluga’s claim that the cognitive origins of the concept of sense require that priority be given to sentences. No version of the context principle for referential significances follows from these claims. Unfortunately, Sluga never says what exactly he takes the context principle to be, whether a doctrine about senses, references, or both. Frege’s original formulation, of course, preceded his making this distinction. So perhaps the best conclusion is that Sluga takes the principle to persist as applying to senses, that is that it is only in the context of a thought that a term or other sub-sentential expression expresses a sense. This seems to be something Frege indeed did not surrender. Such a reading has the additional advantage that the doctrine that concepts are functions from the references of singular terms to truth-values is not incompatible with it.

The final argument fares less well. It is claimed that Frege’s late treatment of real numbers shows that his practice is still in accord with the context principle.<sup>47</sup> Here the point seems to be that the real numbers are given contextual definitions. Such an argument would be relevant to a context principle applying to reference rather than

senses, since Frege does not pretend to specify the senses of numerical expressions in his formal definitions. But the definition of real numbers he offers is of just the same form as the *Grundgesetze* definition of natural numbers. If this style of definition does exhibit commitment to a form of the context principle, that case should be argued for the more central and important case of natural numbers. It is not clear how such an argument would go.

### III

One of the themes around which Sluga usefully arranges his presentation of Frege's development is that of the pursuit of the definition of purely logical objects. The reason offered for the somewhat misleading order of presentation pursued in ÜSB, which seems to give pride of place to singular terms rather than sentences, is that the road from the *Grundlagen* account of numbers to that of the *Grundgesetze* needed to pass through a more thorough understanding of identity claims. Sluga is quite clear that for Frege, beginning with the *Grundlagen*, the only concept we have of an *object* is as that which determines the semantic significance of a singular term. For an expression to play the semantic role of a singular term is for it to make a certain contribution to the inferential potential of sentences containing it, a contribution which is constituted by the appropriate (truth preserving) substitutions which can be made for that expression. The substitution inference potential of a singular term is in turn codified in the endorsed identity claims involving that term. That what we mean by 'object' is according to Frege exhausted by our conception of that the recognition of which is expressed in identity claims in virtue of their licensing of intersubstitution is one genuinely transcendental element in his thought about which Dummett, Sluga, and Bell agree.

In the *Grundlagen* Frege argued that according to this criterion, number-words are singular terms, so that if statements about them are ever objectively true or false they must be so in virtue of properties of the objects which are identified and individuated in assertions of numerical identities. The logicist thesis that the truths of mathematics are derivable from the truths of logic by logical means alone accordingly entails that numbers are purely logical objects, in the sense that the identities which express the recognition and individuation of these objects are themselves logical truths. Sluga's ingenious suggestion is that Frege's concern in *ÜSB* with the nature of *synthetic* or potentially knowledge-extending identities specifying ordinary objects should be understood as a stage in the working out of his mature account of analytic (logically true) identities required for the adequate specification of the logical objects treated in the *Grundgesetze*. The specific interpretive use to which Sluga puts this general insight is hard to warrant, however .

For he claims that the difference between these two sorts of identities resides in the fact that the identities by which logical objects are identified and individuated express coincidence not just of reference, but also of sense.<sup>48</sup> It is not clear what reasons there are to accept this reading, nor what interpretive advantages would accrue from doing so. For Frege explicitly affirms on a number of occasions that the two expressions '2<sup>2</sup>' and '2 + 2' express different senses. And he seems committed to this view by structural principles of his approach, in particular by the compositionality principle as it applies to senses. Different function-expressions appear in these two complex designations, and the senses of components are parts of the senses of complexes containing them. Nor does the fact that such identities are to be logically true entail that they express identities of sense

rather than merely of reference. Identity of sense would of course be sufficient for identity of reference. But we are often told that logic need be concerned only with truth and reference, and Frege's view seems to be that it can be logically true that two different senses determine the same reference.

This mistake aside, Sluga's tracing of the development of Frege's attempts to define abstract objects of the sort instantiated by logical objects is a valuable contribution, and raises issues of the first importance for our understanding of the constraints on interpretations of Frege's technical concepts. The story begins with the second definition of number which Frege tries out in the *Grundlagen*. It states that two concepts have the same number associated with them if and only if the objects those concepts are true of can be correlated one-to-one.<sup>49</sup> He rejects such a definition as inadequate to specify numbers as objects, on the grounds that it will not determine whether, for example, Julius Caesar or England are identical to any number. Such a definition settles the truth values of identities (and hence the appropriateness of substitutions) only for terms which are the values for some argument expression of the function-expression "the number of the concept . . .". This procedure would be legitimate only if we had independently defined the concept (function from terms to truth values) *number* signified by this function. expression. But it is not possible simultaneously to specify that function and the objects for which it yields the value True. If objects had been specified by this definition, then there would be a fact of the matter as to whether Julius Caesar was one of them. But the definition does not settle this issue either way. On the basis of this objection, Frege motivates his third and final definition of numbers, considered below.

Sluga traces through the later works Frege's efforts to clarify the specification of numbers in such a way that it will not be subject to this objection, culminating in the *Grundgesetze* account of courses of values. Given the centrality to Frege's project of producing an adequate definition of number this progress is of interest for its own sake. But the task of responding to the objection to the second *GL* definition of number is made especially urgent for interpreters of Frege by a consideration which Sluga does not mention. For the specifications of the abstract objects in terms of which Frege's semantic analysis proceeds (e.g. sense, reference, thought, truth-value) are of the same objectionable form as the second *GL* definition of number. Nothing we are ever told about the senses of singular terms or sentences, for instance, settles the question of whether Julius Caesar can be such a sense. Though this may seem like a question of no interest, some interesting questions do take this form. For in interpreting the notion of sense one is concerned *both* with subdividing the explanatory functional role played by the concept (as exhibited in the discussion of Bell) *and* with the possibility of identifying senses with things otherwise described—for example, the uses of expressions, sets of possible worlds, mental representations. Frege himself addresses such issues when he denies that the senses of sentences are to be identified with ideas in people's minds. How is the identity he wishes to deny given a sense?

All that is given is a criterion determining when the senses associated with two expressions are the same (namely if they are intersubstitutable without change of cognitive value—*Erkenntniswerte*). If something is not specified as the sense associated with an expression (compare: number associated with a concept) its identity or nonidentity with anything which has been so given is entirely undetermined. Frege's

procedure for introducing his technical concepts such as sense is invariably to attempt to specify *simultaneously* a realm of abstract semantic interpretants *and* a function which assigns a member of this realm to each expression.

We are, for instance, to associate truth-values with sentences. But we are told only that the truth-value associated with  $p$  is the same as the truth-value associated with  $q$  just in case for no occurrence of  $p$  (either as a free-standing sentence or as a component in a more complex sentence) can a good inference be turned into a bad one by substituting  $q$  for that occurrence of  $p$  (reading the principle that good inferences never take true premises into conclusions that are not true as defining truth-values in terms of the goodness of inferences). Even conjoining such a specification with the stipulation that the truth-value associated with the sentence ' $2 + 2 = 4$ ' is to be called 'the True' does not settle the question of whether Julius Caesar is that truth-value. He had better not be, for if the logicist program of *GG* is to be successful, truth-values must be definable as purely logical objects. The current question is how the identity which is denied here is given a sense so that something could count as justifying that denial. The functions which associate the various kinds of semantic significances with expressions are always of the form:  $f(x) = f(y)$  iff  $R(x, y)$ , where  $x$  and  $y$  range over expressions, and  $R$  is some relation defined in terms of the inferential potentials of those expressions. These are exactly the kind of definition Frege found wanting in *GL*.

Seeing how Frege believes he can overcome the objectionable indeterminateness of concepts such as that determined by the second *GL* definition of number is thus a matter of considerable importance for the appraisal of his success in specifying his own technical concepts, as well as for the narrower project of introducing numbers as logical

objects. The third and final definition of number which Frege offers in *GL* is: “the Number which belongs to the concept  $F$  is the extension of the concept ‘equal (*Gleichzahlig*) to the concept  $F$ ’”.<sup>50</sup> The number three is thus identified with the extension of the concept, for example, “can be correlated one-to-one with the dimensions of Newtonian space”. This definition does not have the form Frege had objected to. However, it essentially involves a new concept, ‘extension’, which has not previously appeared in *GL*, nor indeed anywhere else in Frege’s writings. In a footnote to the definition, Frege says simply “I assume that it is known what the extension of a concept is.” Sluga points out that this definitionally unsatisfactory situation is not remedied in the remainder of the book. The result is scarcely up to the standards of definition to which Frege held others and himself. The project of *GL* could not be counted a success until and unless it could be supplemented with an account of the extensions of concepts.

Six years later, in ‘Funktion und Begriff’, Frege offers such an account. The general notion of a function is explicated, and concepts are defined as functions from objects to truth-values. The extension of a concept is defined as the ‘course of values’ (*Wertheverlauf*) of that function. This is the first appearance of the concept of a course of values. Since extensions are reduced to them, the residual definitional burden bequeathed by *GL* is put off onto this new concept. What Frege tells us here is just that the course of values associated with function  $F$  is the same as the course of values associated with function  $G$  just in case for every argument the value assigned to that argument by  $F$  is the same as the value assigned to it by  $G$ . The trouble with such a stipulation, as Sluga says, is that it has exactly the objectionably indeterminate form of the second *GL* definition of number which it is invoked to correct. Frege wants to associate with each function a new

kind of object, a course of values. This domain of objects and the function which assigns one to each function are introduced simultaneously. The result is that it has not been determined whether Julius Caesar is the course of values of any function. A given course of values has only been individuated with respect to other objects specified as the courses of values associated with various functions. In sum, the courses of values in terms of which the extensions of concepts are defined suffer from exactly the defect of definition which extensions of concepts were introduced to rectify or avoid.

In the *Grundgesetze* when courses of values are introduced this difficulty is explicitly acknowledged and described in the same terms used to raise the original objection in *GL* (though without reference to the earlier work). Frege introduces the same principle for determining when the courses of values of two functions are identical, and then points out that such a principle cannot be taken to determine any objects until criteria of identity and individuation have been supplied with respect to objects which are *not* given as courses of values. He proposes to supplement his definition so as to satisfy this demand. His proposal is that for each object not given as a course of values it be *stipulated* to be identical to an *arbitrary* course of values, subject only to the condition that distinct objects be identified with distinct courses of values.

Frege expresses the function which assigns to each function an object which is its course of values by means of an abstraction operator binding a Greek variable. The course of value of a function  $F$  is written as  $\delta(F\delta)$ . Axiom V of the *Grundgesetze* tells us that:

$$(a) \delta(F\delta) = \alpha (G\alpha) \text{ iff } (\forall x) [Fx \Leftrightarrow Gx].$$

Frege recognizes that this principle alone does not suffice to determine the identity of objects which are courses of values. To show this he points out that if  $X$  is a function which yields distinct values if and only if it is applied to distinct arguments (what we may call an “individuation preserving” function), then:

$$(a') X('δ(Fδ)) = X('α (G α )) \text{ iff } (\forall x) [Fx \Leftrightarrow Gx]$$

without its having been settled for instance whether

$$(a'') X('δ(Fδ)) = 'α (G α )$$

for any  $F$  and  $G$  (including the case in which  $F = G$ ). The by now familiar point is that (a) only determines the truth-values of *homogeneous* identities, those both terms of which are of the form  $'δ(Fδ)$ . And (a') only determines the truth-values of identities which are homogeneous in that both terms have the form  $X('δ(Fδ))$ . But (a'') asks about *heterogeneous* identities, whose terms are of different forms. Another identity which is heterogeneous and whose truth-value is accordingly not settled by principle (a) is Julius Caesar =  $'δ(Fδ)$ .

To fix up this indeterminateness, which would result from taking Axiom V alone as the definition of courses of values, Frege proposes to supplement it by stipulating the truth-values of the heterogeneous identities. Actually, he is required to specify the inferential behavior of course of value expressions in all contexts in which they can appear. In Frege's terminology such contexts are functions, so this requirement is equivalent to the demand that it be determined for every single-argument function-expression what value is designated by the substitution of any course of values expression in its argument place. Doing so will then determine all of the properties of the objects designated by expression of the form  $'δ(Fδ)$ , for those properties just are concepts,

that is, functions whose values are truth-values. Among those properties are individuating properties, the facts corresponding to identity contexts involving course of values expressions. Thus the *Grundlagen* requirement that to introduce a new set of objects one must settle all identities involving them is in the *Grundgesetze* motivated by the omnicontextual condition. (It is worth noticing, as Sluga points out, that there is an endorsement of a strong context principle in Frege's claim that what it is to have introduced expressions of the form ' $\delta(F\delta)$ ' as the names of definite objects is for the truth-values of all sentential contexts in which those expressions can be substituted to have been settled.) In fact, in the spare environment of *GG* it turns out that it is not only necessary to settle the truth-values of all identities involving course of value expressions in order to satisfy the omnicontextual requirement, but sufficient as well.

Indeed, in the system of the *Grundgesetze* at the time courses of values are introduced the only objects already defined are the two truth-values, and so the only heterogeneous identities Frege explicitly addresses are those involving a course of values and a truth-value. But he must justify the *general* procedure of stipulating truth-values for heterogeneous identities, and not just his application of it. For if he does not, then the *GG* definition of number will still be open to the objection to the second *GL* account of number (that it has not been settled whether Julius Caesar is one) which drove him to define the extensions of concepts and hence courses of values to begin with. Indeed, the concept "logical object" will not have been defined if it has not been settled whether Julius Caesar is one. Further, as we have seen, Frege's own definitions of his technical terms in general suffice only to determine the truth-values of homogeneous identities, for example, identities of two truth-values, or two senses, or two references, but not the

heterogeneous identities which would be required to make the claim that Julius Caesar = the *Bedeutung* of the expression ‘Julius Caesar’, or that a certain linguistic role *is* the sense of some expression.

In particular, Frege’s substitutional-inferential methodology determines only the nonrelational sense of ‘*Bedeutung*’, according to which expressions are sorted into substitutional equivalence classes as having the same *Bedeutung*. For Frege to add to this determination of homogeneous identities (both of whose terms are of the form “the *Bedeutung* of the expression *t*”) the relational sense of reference in which these *Bedeutungen* are *identified* with objects suitably related to all and only the members of the nonrelational substitutional equivalence class of expressions is precisely to stipulate the truth-values of the *heterogeneous* identities. The question of whether such a procedure can be justified on Frege’s own terms is thus exactly the question of whether the two notions of *Bedeutung* can be made into “two aspects of *one* notion” as Dummett claims and Frege is committed to, or whether they are simply conflated without warrant, as Bell claims. Following Sluga’s development of Frege’s attempted definition of terms which refer to logical objects thus leads to the argument which must justify the identification of the things playing the two explanatory roles which Bell has shown must be distinguished under the heading “*Bedeutung*”.

In Section 10 of *GG* Frege offers his justification of the procedure of stipulating the heterogeneous identities, in an argument which Currie has called “brilliantly imaginative”.<sup>51</sup> The argument is a difficult one, and we shall have to examine it with some care. What is to be shown is that it is legitimate to stipulate (a) above, determining

the homogeneous identities involving courses of values, together with the following stipulation for heterogeneous identities:

$$(b) \quad \text{'}\tau(L\tau) = t_1 \text{ and '}\sigma(M\sigma) = t_2$$

where  $t_1 \neq t_2$  and  $(\exists x)(Lx \neq Mx)$ .  $L$  and  $M$  are to be arbitrary functions, and  $t_1$  and  $t_2$  are terms which are not of the form  $\text{'}\alpha (F\alpha)$ . For the purposes of the GG argument, the terms in question are “the True” and “the False”. In the context of Sluga’s point that Frege’s defense of his own view against his objection to the second attempted definition of number in *GL* must be traced through the account of extension to the account of courses of values, it will be worth keeping in mind that for this purpose the argument must apply equally to the case in which  $t_1$  is “Julius Caesar” and  $t_2$  is “England”. To emphasize this requirement, the exposition of Frege’s argument which follows will use those values for  $t_1$  and  $t_2$  rather than the truth-values which Frege employed. In any case the point is that distinct objects which are *not* given as courses of values are stipulated to be identical to the courses of values a like number of arbitrary distinct functions. The task is to show that such a stipulation is legitimate.

The strategy of the argument is to *construct* a domain of objects of which (a) and (b) can be *proven* to hold. To start, suppose it has been stipulated that:

$$(c) \quad \sim\eta(F\eta) = \sim\gamma(G\gamma) \text{ iff } (\forall x) [Fx \Leftrightarrow Gx],$$

that is, we stipulate the homogeneous identities for terms of the form  $\sim\eta(F\eta)$ , where the function which associates objects so denominated with functions  $F$  is unknown except that principle (c) holds. As was pointed out above by means of (a’) and (a’”), the fact that both (a) and (c) hold does not in any way settle the heterogeneous identities one of whose terms is a course of values and the other of which is of the form  $\sim\eta(F\eta)$ . The next step is

to use the arbitrary distinct functions  $L$  and  $M$  of (b) to construct an individuation preserving function  $X$  as above. The function  $X$  is defined by five clauses:

- (1)  $X(\text{Julius Caesar}) = \sim\eta(L\eta)$
- (2)  $X(\sim\eta(L\eta)) = \text{Julius Caesar}$
- (3)  $X(\text{England}) = \sim\gamma(M\gamma)$
- (4)  $X(\sim\gamma(M\gamma)) = \text{England}$
- (5) For all other  $y$ ,  $X(y) = y$ .

The function  $X$  is constant except when it is applied to either the two objects which are not specified as the result of applying  $\sim$ -abstraction to some function (Julius Caesar and England, or the True and the False) or to the result of applying  $\sim$ -abstraction to the arbitrarily chosen functions  $L$  and  $M$ . In these special cases, the function  $X$  simply permutes the distinguished values.

$X$  is constructed to be individuation preserving, so that a correlation is preserved between distinctness of its arguments and distinctness of its values. It follows then that:

- (d)  $X(\sim\eta(F\eta)) = X(\sim\gamma(G\gamma \text{ iff } (\forall x)[Fx \iff Gx])$ .

In these terms we could now *define* the course of values notation (which has not previously appeared in this argument) by agreeing to let:

- (e)  $\alpha(F\alpha) =_{df} X(\sim\eta(F\eta))$  for all functions  $F$ .

Given the definition (e) and the truth of (d), principle (a) for courses of values follows immediately. The truth of (d), as we have seen, follows from (c), together with clauses (1)-(5) defining the function  $X$ . But Clauses (2) and (4) of that definition, together with (e), entail principle (b) concerning courses of values (with the substitution of Julius Caesar for  $t_1$  and England for  $t_2$ ). Thus given only the homogeneous identities

in (c) we have constructed courses of values in such a way that their homogeneous identities in (a) can be shown to hold *and* in such a way that heterogeneous identities can be *proven* for two of them, since ‘ $\alpha(L\alpha) = \text{Julius Caesar} (= X(\sim\eta(L\eta))$  and ‘ $\delta(M\delta) = \text{England} (= X(\sim\gamma(My)))$ ). The legitimacy of stipulating heterogeneous identities in the context of a principle determining homogeneous ones has been shown by reducing the questionable stipulation to the composition of two obviously acceptable forms of stipulation: the specification of the values which the function  $X$  is to take for various arguments (in particular in Clauses (2) and (4)), and the introduction of the expression “ $a(Fa)$ ” (previously without a use) as a notational abbreviation of “ $X(\sim\eta(F\eta))$ ”.

This imaginative argument is Frege’s ultimate response defending his account of number and of logical objects generally against the objections he had raised but not answered in the *Grundlagen*. Seen in that context, the argument is fallacious. The problem concerns the extremal Clause (5) of the definition of the individuation preserving function  $X$ . If that clause is expanded to make explicit what is contained in the condition “for all other  $y$ ” it becomes:

$$(5') \quad (\forall y)[(y \neq \text{Julius Caesar} \ \& \ y \neq \sim\eta(L\eta) \ \& \ y \neq \text{England} \ \& \ y \neq \sim\gamma(My)) \Rightarrow X(y) = y].$$

It may then be asked whether it is appropriate at this point in the argument to make use of a condition such as  $y \neq \sim\gamma(My)$ . If the term substituted for ‘ $y$ ’ is also represented as the product of applying  $\sim$ -abstraction to some function, then clause (c) will settle the truthvalue of the resulting identity. For it settles just such homogeneous identities. But what of the case in which the identity is *heterogeneous*? *All* that has been fixed concerning  $\sim$ -abstraction is principle (c), which says nothing about such identities.

Indeed, the whole strategy of the argument depends upon starting from a specification of purely homogeneous identities with one sort of abstractor ( $\sim$ ) and using the function  $X$  to construct an abstractor ( $\cdot$ ) for which the heterogeneous identities are specified. Nothing which has been said, or, given the strategy just indicated, *could* be said, settles a truth-value for heterogeneous identities such as

$$(f) \quad \text{Julius Caesar} = \sim\gamma(M\gamma)$$

and

$$(g) \quad \text{England} = \sim\eta(L\eta).$$

For all that principle (c) concerning  $\sim$ -abstraction and the distinctness of the functions  $L$  and  $M$  settle, (f) could be true and (g) false. Given the truth of (f), substituting in Clause (4) would yield that  $x(\text{Julius Caesar}) = \text{England}$ , and so by clause (1) that  $\text{England} = \sim\eta(L\eta]$ , that is, that (g) is true. So the definition of  $X$  presupposes valuations for heterogeneous identities which it is in no way entitled to.

Matters are just as bad if we consider some other object, say the direction of the Earth's axis (also discussed in *GL*). It has nowhere been determined whether it is identical to  $\sim\eta(L\eta]$  and so falls under Clause (2), or identical to  $\sim\gamma(M\gamma)$  and so falls under Clause (4), or to neither and hence falls under Clause (5). The definition of  $X$ , in terms of which it is to be shown acceptable to stipulate heterogeneous identities for  $\sim$ -abstraction, is well-formed only if the heterogeneous identities involving  $\sim$ -abstraction have already been settled. They have not been settled. Further, to add to the argument the assumption that truth-values for such heterogeneous identities involving expressions of the form  $\sim\delta(F\delta)$  have been settled is to assume exactly what the argument as a whole is supposed to show, namely that such matters are open for stipulation in the first place

(so long as suitable care is taken to match distinct objects with the result of abstracting distinct functions). If more is supposed about  $\sim$ -abstraction than principle (c) fixing homogeneous identities, the question will be begged. And without some supposition about heterogeneous identities the argument does not go through.

The intent of the offending extremal clause is to deal with all objects which can be represented by expressions of the form  $\sim\delta(F\delta)$ , where  $F \neq L$  and  $F \neq M$ . Distinct objects not so representable are each to be dealt with by a pair of clauses, letting the function  $X$  permute them with the result of abstracting from corresponding arbitrarily chosen distinct functions. There is nothing in general wrong with such a definitional strategy. It may not be used in the context of this argument, however. The distinction between the cases which are to be dealt with by paired specific stipulations and those which remain to be dealt with by the extremal stipulation cannot be made precise without begging the question. For that distinction corresponds to the distinction between heterogeneous identities and homogeneous ones, in the sense of stipulations for objects not representable by expressions of the form  $\sim\delta(F\delta)$  and those which are so representable. This distinction is not one which a principle like (c) specifying the purely homogeneous identities permits us to make, and we are entitled to presuppose no more than such a principle. Put otherwise, the form of definition essentially requires that there be a pair of specific clauses dealing with *every* object whose individuation with respect to the results of applying  $\sim$ -abstraction to functions has not been settled by principle (c). But this class of objects cannot be described or specified in the terms permitted for the definition if it is to play its appointed role in the larger argument.

The only way in which this situation might be remedied would be if there were some property available which could be independently appealed to in order to distinguish the two kinds of cases. Thus if to (c) were added:

$$(c') \quad (\forall y)[P(y) \Leftrightarrow (\exists F)(y = \sim\delta(F\delta))]$$

then the extremal clause in the definition of  $X$  could be amended to

$$(5'') \quad (\forall y)[P(y) \ \& \ y \neq \sim\eta(L\eta) \ \& \ y \neq \sim\gamma(M\gamma) \Leftrightarrow X(y) = y]$$

In the context of (c'), (5'') will have the desired effect of applying only to objects which can be designated by expressions of the form  $\sim\delta(F\delta)$ , where  $F \neq L$  and  $F \neq M$ . More important, (c') would ensure that the identities in (5'') are homogeneous with respect to  $\sim$ -abstraction, and hence have had their truth-values settled by (c). It was the failure to ensure the homogeneity that was responsible for the inadequacy of the original definition of  $X$ .

The trouble with this way out is that no such independently specifiable property is available. Already in the *Grundlagen* Frege had pointed out that the account of when the numbers associated with two concepts were identical (settling identities homogeneous with respect to the form: the number of the concept  $F$ ) could be defended against his objection if the concept “. . . is a number” were available. For then the truth-values of the heterogeneous identities (such as those involving Julius Caesar) could be settled by specifying that for any  $t$ , if  $t$  is not a number, then it is not identical to the number of any concept. But the problem the desired definition was to solve was precisely that of specifying the concept “. . . is a number”, as the current task is to specify the concept “. . . is a course of values”. It would be circular to use for the property  $P$  “. . . is an  $x$  such that there is an  $F$  such that  $x = \sim\delta(F\delta)$ ”. For that would precisely presuppose that the

heterogeneous identities have somehow already been settled, rather than independently settling them. Nor could some property such as “. . . is not in the causal order” be used, for there are other logical objects (such as the True and the False) whose individuation with respect to objects specified by  $\sim$ -abstraction has not been determined. Nor in any case would such a property be available to a logicist.

Frege’s argument does not work, then, and it cannot be made to work. If the *Grundgesetze* is meant to offer an account of number which will meet the demands set by the *Grundlagen*, then it is a failure by Frege’s own standards. Further this failure is not a matter of the inconsistency of the later system. Although Axiom V is the culprit in both cases, it is different features of that principle which are found objectionable in the two cases. The current complaint is that settling the truth values of the homogeneous identities alone, as that principle does, is definitionally too weak to meet the requirements imposed by the discussion of *GL*. Those demand the justification of the stipulative extension of the definition to heterogeneous identities. That it leads to inconsistency, on the other hand, shows that that Axiom is inferentially too strong. Putting aside the question of inconsistency which makes the claim counterfactual, even if the account of courses of values in *GG* were technically adequate, it would not be philosophically adequate as a specification of its objects and concepts. For it has not settled whether Julius Caesar is the number three, nor shown that stipulating an answer in the case of logical objects such as the truth-values is a legitimate procedure. Nor can this be shown with the materials at hand.

I take it that this definitional inadequacy has not been remarked upon for two connected reasons. In the purely technical context of the *Grundgesetze* the stipulation of

the two heterogeneous identities concerning the truth-values and arbitrary distinct courses of values is in fact perfectly acceptable. Further, provided that it is stipulated that neither of the truth-values is identical to the result of applying  $\sim$ -abstraction to any function, Frege's argument shows that his procedure is in order. It is only in the larger philosophical context provided by Sluga's historical tracing of the stages in Frege's development of an answer to his own objections to the second attempted definition of number in *GL*, from the invocation of the extension of a concept in the third and final *GL* definition, via the reduction of concepts to a special kind of function and of extensions to courses of values in 'Funktion und Begriff', to the final attempt to define courses of values adequately in the early sections of the *Grundgesetze* that it can be seen that satisfying the purely technical constraints will not suffice to render the definition of courses of values (and hence of logical objects generally) adequate by the philosophical standards Frege has insisted upon.

But the result is significant not only for the appraisal of the success in its own terms of Frege's account of the logical objects which were his explicit subject matter in *GG*. For as we have seen, the technical philosophical concepts Frege developed to use in that discussion, such as reference, and sense, and truth-value, are all given the same form of definition as courses of values are, which individuates them only homogeneously. Thus ". . . we cannot say *what* the sense of an expression is. The closest we may approach to this is to say that the sense of a given expression  $E_1$  is the same as the sense of another expression,  $E_2$ ."<sup>52</sup> It follows that so far as *interpretation* (rather than further development) of Frege's concept of sense is concerned, one can *only* subdivide the explanatory roles played by his concept, but cannot identify anything as playing those

roles. Thus it is legitimate and valuable to distinguish the cognitive role from the semantic role played by senses, or sense as content from sense as character, or input and output senses as Bell does. But to entertain hypotheses about whether thoughts are mental pictures (as Frege did by denying this) or sets of possible worlds, or denizens of some extra-causal realm is to consider claims which have been given no sense by Frege's purely homogeneous specification of the entities in question. Truth values are similarly immune from heterogeneous identification, from identification in any other form than as the truth-value associated with some expression.

Probably most important is the case of singular term reference. Here Frege tried explicitly to supplement the purely homogeneous sorting into semantic equivalence classes of the reference associated with various expressions (the nonrelational sense of '*Bedeutung*') with the stipulation of heterogeneous identities involving the references of expressions and ordinary objects. In accord with his inferential/substitutional methodology, these stipulations are grounded in the intersubstitutability for all terms *t* of the term itself and the expression 'the *Bedeutung* of *t*'. Bell has shown how much of Frege's conceptual scheme depends upon the assumption that such heterogeneous identities are determined (and hence a relational sense of reference applies) for other parts of speech, given only the determination of the homogeneous identities (settling a non-relational sense of reference) which is all that is available for expressions of these other categories. Pursuing further a line of thought Sluga initiated has shown that this assumption is indeed unwarranted, and that even Frege's attempted stipulation of coincidence of relational and nonrelational senses of 'reference' in the case of singular terms has not been justified by Frege's own standards. Thus extending Sluga's argument

permits better understanding of the philosophical status of Frege's technical concepts in general, and in particular of the two sides of the concept of reference which Bell, following Dummett, has so usefully distinguished.

<sup>1</sup> See Hermes, H., Kambartel, F., and Kaulbach, F. (eds.), *Gottlob Frege: Posthumous Writings*, transl, by P. Lond and R. White, [University of Chicago Press, Chicago, 1981]. Hereafter [Frege *Posthumous Writings*].

<sup>2</sup> Dummett, Michael, *Frege: Philosophy of Language* [Harper and Row, New York, 1973]. Hereafter [Dummett *FPL*]

<sup>3</sup> Dummett, Michael, *The Interpretation of Frege's Philosophy* [Harvard University Press, Cambridge, 1981]. Hereafter: [Dummett, *IFP*].

<sup>4</sup> [Dummett, *IFP*] pp. xii-xvi.

<sup>5</sup> The most influential proponents of the view were Grossmann, R., 'Frege's Ontology', *Philosophical Review* 70, 1961, pp. 23-40, and Marshall, W., 'Frege's Theory of Functions and Objects', *Philosophical Review* 62, 1953, pp. 347-390, and Marshall, W., 'Sense and Reference: A Reply', in Klemke, E., *Essays on Frege* [University of Illinois Press, Urbana, 1968].

<sup>6</sup> p. 65 in Geach, P. and Black, M. (eds.), *Philosophical Writings of Gottlob Frege*, [Blackwells, Oxford 1970]. Hereafter [Geach and Black *PWGF*].

<sup>7</sup> Bell, David, *Frege's Theory of Judgement* [Oxford University Press, Oxford, 1979]. Hereafter [Bell, *FTJ*].

<sup>8</sup> As in the title of Dummett's book, "*Frege: Philosophy of Language*" [Dummett *FPL*]

<sup>9</sup> pp. 476-95. in [Dummett *IFP*].

<sup>10</sup> In Section 3 of *BGS*, reprinted in [Geach and Black *PWGF*], Frege says that the *begriffliche Inhalt* of two judgements is the same just in case "all inferences which can be

drawn from the first judgement when combined with certain other ones can always also be drawn from the second when combined with the same other judgements.”

<sup>11</sup> pp. 9-46 in [Frege *Posthumous Writings*].

<sup>12</sup> For instance by Resnik in Resnik, Michael, ‘The Context Principle in Frege’s Philosophy’, *Philosophy and Phenomenological Research* 27, 1967, pp. 356-365; and Resnik, Michael ‘Frege’s Context Principle Revisited’, included in Schirn, M. (ed.), *Studien zu Frege*, Vol. III [Frommann-Holzboog, Stuttgart and Bad Cannstatt, 1967], pp. 35-49. See also Angelelli. I., *Studies on Gottlob Frege and Traditional Philosophy*, [D. Reidel, Dordrecht, 1967].

<sup>13</sup> pp. 139-40 in [Bell, *FTJ*].

<sup>14</sup> I have my say in, ‘Asserting’, *Noûs* 17, 1983. pp. 637-650.

<sup>15</sup> p. 42 [Bell, *FTJ*].

<sup>16</sup> pp. 478-9 [Dummett, *IFP*].

<sup>17</sup> In Tugendhat, E., ‘The Meaning of “Bedeutung” in Frege’, *Analysis* 30, 1970, 177-189.

<sup>18</sup> p. 479 [Dummett, *IFP*].

<sup>19</sup> I have argued that a purely intralinguistic anaphoric account of such facts can be offered by construing ‘refers’ and its cognates as complex pronoun-forming operators, in ‘Reference Explained Away’, *Journal of Philosophy* 84, 1984, pp. 769-792.

<sup>20</sup> Saul Kripke in, ‘Naming and Necessity’, and Hilary Putnam in ‘The Meaning of Meaning’, both in Harman, G. and Davidson, D. (eds.), *Semantics of Natural Language* [D. Reidel, Dordrecht, 1972].

<sup>21</sup> Most prominently, John Perry in ‘Frege on Demonstratives’, *Philosophical Review* 86, 1977, pp. 474-497, and David Kaplan in ‘The Logic of Demonstratives’, in Uehling, T., Wettstein, H., and French, P. (eds.), *Contemporary Perspectives on Philosophy of Language, Midwest Studies in Philosophy* [University of Minnesota Press, Minneapolis, Minnesota, 1978] and Kaplan, David *Demonstratives*, 1980 John Locke Lectures [Oxford University Press, 1984].

<sup>22</sup> p. 112 in [Bell, *FTJ*].

<sup>23</sup> p. 115 in [Bell, *FTJ*].

<sup>24</sup> p. 51 in [Bell, *FTJ*].

<sup>25</sup> p. 64 in [Bell, *FTJ*].

<sup>26</sup> p. 65 in [Bell, *FTJ*].

<sup>27</sup> First in [Dummett *FPL*], pp. 293-4, then at greater length as chapter 13 of [Dummett, *IFP*]. Citations here from p. 251 of the latter.

<sup>28</sup> In Geach, Peter, ‘Review of Dummett’s *Frege: Philosophy of Language*’, *Mind* 85, 1975, 436-449.

<sup>29</sup> pp. 251-2 [Dummett *FPL*].

<sup>30</sup> p. 119, [Frege *Posthumous Writings*]. Bell’s other quotations are in p. 72 of [Bell, *FTJ*].

<sup>31</sup> pp. 74-8 of [Bell, *FTJ*].

<sup>32</sup> See Sluga, Hans, *Gottlob Frege* [Routledge and Kegan Paul, London, 1980]. Hereafter [Sluga *GF*].

<sup>33</sup> Michael Dummett in 'Frege, Gottlob', in Edwards, P. (ed.), *Encyclopedia of Philosophy* [The Macmillan Company & The Free Press, New York/Collier -Macmillan Ltd., London, 1967], Vol. 4, p. 225, quoted by Sluga at p. 8 of [Sluga *GF*].

<sup>34</sup> In Dummett's defense it should be said that in the final chapter of [Dummett *FPL*], the main historical significance of Frege's work is taken to be precisely his anti-empiricist and anti-psychologist shifting of concern from the acquisition of concepts to what such mastery consists in—from how the cognitive trick is performed (e.g., by material beings of our sort) to what counts as performing it. The injudicious invocation of a dominant Hegelianism as the psychologistic culprit is explicitly made subsidiary to this central point.

<sup>35</sup> p. 59 [Sluga *GF*].

<sup>36</sup> Michael Dummett, in 'Frege as Realist', *Inquiry* 19, 1976, pp. 476-485.

<sup>37</sup> Bierich, M., *Freges Lehre von dem Sinn und der Bedeutung der Urteile und Russells Kritik an dieser Lehre* [Dissertation, Hamburg, 1951].

<sup>38</sup> pp. 53, 192 [Sluga *GF*].

<sup>39</sup> pp. 59-60 [Sluga *GF*].

<sup>40</sup> pp. 44-5, and 106 [Sluga *GF*].

<sup>41</sup> p. 60 [Sluga *GF*].

<sup>42</sup> p. 60 [Sluga *GF*].

<sup>43</sup> p. 91 [Sluga *GF*].

<sup>44</sup> p. 181 [Sluga *GF*].

<sup>45</sup> p. 95 [Sluga *GF*].

<sup>46</sup> p. 253 [Frege *Posthumous Writings*].

<sup>47</sup> p. 134 and Note 21 to chapter 4 [Sluga *GF*].

<sup>48</sup> E.g. at p. 156 [Sluga *GF*].

<sup>49</sup> Sections 62, 63 of Frege, Gottlob, *Grundlagen der Arithmetik*, transl. by J. L. Austin [Northwestern University Press, Evanston, Illinois, 1967]. Hereafter [Frege *GL*].

<sup>50</sup> Section 68 of [Frege *GL*]. Equality of concepts in the sense invoked here has been defined as obtaining iff the objects of which the concepts are true can be put into one-to-one correspondence.

<sup>51</sup> p. 69 in Currie, G., *Frege: An Introduction to His Philosophy* [Barnes and Noble, Totowa, New Jersey, 1982].

<sup>52</sup> Bell in [Bell, *FTJ*] p. 55 [emphasis in original]. He goes on to point out an analogy with the Fregean concept of concept reference: “concept words refer, but we cannot stipulate *what* it is they refer to”. But in this case the reasons are purely substitutional, since expressions like “the concept *horse*” will never be intersubstitutable with predicative function-expressions. This shows that all heterogeneous identities involving function-expressions on one side and singular terms on the other must be false.