Why Must Incompatibility Be Symmetric?

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Abstract

Why must incompatibility be symmetric? A odd question, but recent work in the semantics of non-classical logic, which appeals to the notion of incompatibility as a primitive and defines negation in terms of it, has brought this question to the fore. Francisco Berto proposes such a semantics for negation argues that, since incompatibility must be symmetric, double negation introduction must be a law of negation. However, he offers no argument for the claim that incompatibility really *must* be symmetric. Here, I provide such an argument, showing that, insofar as we think of incompatibility in normative pragmatic terms, it can play its basic pragmatic function only if it is symmetric. The upshot is that we can vindicate Berto's claim about the symmetry of incompatibility but only if we, *pace* Berto, think about incompatibility, in the first instance, as a *pragmatic* relation between *acts* rather than a *semantic* relation between *contents*.

Key Words: Incompatibility, Negation, Non-Classical Logic

0 Introduction

A car's being red is incompatible with its being blue, and, just as well, a car's being blue is incompatible with its being red. Someone's missing the bus is incompatible with their catching it, and, just as well, someone's catching the bus is incompatible with their missing it. Here we have facts consisting in a relation of incompatibility between two states of affairs or event types a and b, and, in both cases, where a is incompatible with b, b is incompatible with a. So,

incompatibility, at least in these cases, is symmetric. But does the concept of incompatibility deployed here, as such, mandate symmetry? Can we rule out the possibility of there being contents, presumably contents quite different than something's being red or someone's missing the bus, that stand in asymmetric incompatibility relations? Until recently, this question had not received much philosophical attention. However a new trend in thinking about negation in terms of incompatibility (Dunn 1993, 1996; Restall 1999, 2000; Berto 2015; Berto and Restall 2019) has brought it to the fore. A result due to Restall (2000) shows that, if we think of negation as defined on the basis of incompatibility, then double negation introduction ($p \vDash \neg\neg p$) is a law of negation just in case incompatibility is symmetric. Berto (2015) argues that, since incompatibility must be symmetric, anything that is a negation operator must satisfy double negation introduction. However, he gives no argument for the claim that incompatibility *must* be symmetric. Neither he nor anyone else in this trend has said anything to positively rule out the possibility of asymmetrically incompatible contents. Here, I will show that we can indeed rule out such contents, but only if we think about incompatibility quite differently than it is has been presumed we ought to think about it.

Drawing from Brandom (1994), I propose that we think of incompatibility, in the first instance, not as an alethic modal relation that obtains between worldly contents such as states of affairs, but a deontic normative relation that obtains between discursive acts, fundamentally, between acts of making claims. Specifically, two claims p and q are incompatible just in case *commitment* to p *precludes entitlement* to q. Once we adopt this Brandomian frame, we can think about the necessary role that incompatibility plays in a discursive practice: it enables speakers to *challenge* the commitments of other speakers by making claims in-

compatible with them. I show, through consideration of a dialogue that imagines the opposite, that, in order to play this challenge function, a symmetric structure of incompatibility is necessary. So, incompatibility, indeed must be symmetric. If we think of semantic contents as conferred by underlying normative practices, as Brandom does, this explains why an incompatibility relation between any two contents must be symmetric. I conclude by considering, in general terms, the methodology of drawing semantic conclusions from pragmatic premises in this way and how it affords us a new way of thinking about and constructing transcendental arguments.

1 Pros Hen Pluralism About Negation

In a recent paper, Berto (2015) advances a thesis that I'll call *pros hen pluralism* about negation.[1] According to this thesis, "not" is said in many ways. When a classical logician says "not p," they mean something different than what the constructive logician means when they say "not p," who, in turn, means something different than what the paraconsistent logician means when they say "not p." This difference in meaning, however, is not mere homonymy. It's not like the difference in the meaning of word "bank," when it is used to talk about a place where money is stored and when it is used to talk about the edge of a river. No, the meanings of "not," as this expression is used by the respective logicians, are systematically related. The relation between these meanings can be articulated by specifying the different "laws of negation" that the respective

[1]The Greek phrase "pros hen" means "in relation to one." The use of this term is, of course, a reference to Aristotle (See *Met.* Γ 2), though one should not take the concept of a "pros hen" relation employed to be identical to that employed by Aristotle. The more familiar name for this view within logic circles is the "Australian Plan" for negation (Meyer and Martin 1986, Berto and Restall 2019), but I've opted for a descriptive name here rather than a geographical one.

logicians accept. Still, insofar as there is something called *negation* that all of these logicians mean, there ought to be some *common core* to the different "negations" that they respectively employ. If we endorse a pros hen pluralism about negation, we should aspire to have a semantics for negation which (1) captures the *core* meaning of "not" shared by all logicians who are genuinely expressing negation when they use the word, and (2) captures the *secondary* meanings of "not," the different meanings accepted by logicians of different stripes, which are derivative on this core meaning. Berto (2015), drawing on work from Dunn (1993, 1996) and Restall (1999), claims to have a semantics that does just that.

The core meaning of "not," according to Berto, is to be understood in terms of the notion of incompatibility, which is treated as primitive for the purposes of laying down the semantics for negation. Defending the appeal to incompatibility as a primitive notion, Berto writes,

> I take (in)compatibility as the primitive twofold notion grounding the origins of our concept of negation and of our usage of the natural language expression 'not'. Explanations stop when we reach concepts that cannot be defined in terms of other concepts, but only illustrated by way of example. A good choice of primitives resorts to notions we have a good intuitive grip of—and this is the case, I submit, with (in)compatibility.
>
> It is difficult to think of a more pervasive and basic feature of experience, than that some things in the world *rule out* some other things; or that the obtaining of this precludes the obtaining of that; or that something's being such-and-such excludes its being so-and-so, (768-769).

The concept of incompatibility appealed to here is, in the first instance, a *material* rather than *formal* notion.[2] It is a relation that obtains between concepts, judgments, properties, states of affairs, and so on, in virtue of their *material content*

[2]See Sellars (1953) for an influential discussion of this distinction.

rather than their *logical form*. For instance, a monochromatic solid's being red is incompatible with its being blue in the sense that its being red *rules out*, *precludes*, or *excludes* its being blue. This relation between its being red and its being blue is not, properly speaking, a *logical* relation between these two contents, like the relation between something's being red and its not being red, but this material incompatibility relation is appealed to in the semantics in order to specify the properly logical incompatibility relation that obtains between a sentence and its formal negation.

On the semantics proposed by Berto (2015), negation is taken to be a modal operator defined on the basis of this notion of incompatibility. Primitive compatibility relations are modeled by way of an accessibility relation between information-containing "states" (which one can hear as shorthand for "states of affairs") that stand in relations of inclusion to one another.[3] We'll say that a state v *includes* a state w, $w \sqsubseteq v$, just in case v contains all of the information in w. We say that a sentence φ *holds* in a state w, by writing $w \Vdash \varphi$. We assume that, if p holds in w, and v includes w, then p holds in v. That is, we assume the following:

Hereditary Condition: If $w \Vdash p$ and $w \sqsubseteq v$, then $v \Vdash p$

We can now define compatibility as an accessibility relation C that satisfies the following constraint:

Backwards: If wCv and $w' \sqsubseteq w$ and $v' \sqsubseteq v$, then $w'Cv'$

[3]These states are sometimes called "worlds," but I avoid that terminology to avoid confusion with the possible worlds of classical semantics, the sort theorized about by Lewis (1986) and Stalnaker (1984), which are maximally determinate ways for the world to be. If one is to employ the terminology of "worlds" here, the "worlds" at issue are best understood as (at least potentially) *partial* worlds, non-maximal states of affairs that may be more or less determinate. Once again, to avoid confusion, I'll just call them "states."

So, if w is compatible with v, w contains all the information in w', and v contains all the information in v', then w' is compatible with v'. Essentially, this says that, if one state is compatible with another, then, if you remove information from these states, the resulting states continue to be compatible; one can only make compatible states incompatible by *adding* information to one, the other, or both of them.[4] Beyond this formal constraint, the notion of (in)compatibility is not defined, but, rather, appealed to as a primitive. The semantics for negation defined on the basis of this notion of incompatibility says that $\neg\varphi$ holds in a state w just in case, for every state v such that w is compatible with v, φ does not hold. That is:

$\mathbf{S}_{\neg}$: $w \Vdash \neg\varphi$ just in case $\forall v(wCv \supset v \nVdash \varphi)$

To consider an example, "¬(a is blue)" holds in a state consisting in a's being red, since every state that this state compatible with—for instance, the state consisting in b's being blue, the state consisting in a's being red and c's being yellow, and so on—is a state in which "a is blue" fails to hold.

On Berto's pros hen pluralism, this semantics is taken to define the *core* meaning of negation. *Secondary* meanings of negation, the different meanings of "negation" as the term is used by our classical, constructive, and paraconsistent logicians, can then be understood in terms of the imposition of restrictions on the inclusion relation or accessibility relation which do not follow from the set up itself. The classical logician imposes restrictions on the inclusion and accessibility relation that make it such that all states must be *complete* and *consistent*. The constructive logician allows states that are *incomplete*, allowing non-trivial inclusion relations between states, such that it is not the case that, for all states w,

[4] It follows from this constraint that the null state, which contains no information, is compatible with every state that is compatible with some state.

$w \Vdash p$ or $w \Vdash \neg p$. The paraconsistent logician allows states that are *inconsistent*, allowing states that are not compatible with themselves, and to which other states still stand in non-trivial inclusion relations, such that it is not the case that, for all states w, if $w \Vdash p$ and $w \Vdash \neg p$, then $w \Vdash q$.[5] Though these logicians differ in taking negation to have different *secondary* meanings, insofar as all of these meanings are understood in terms of the imposition of restrictions on the models that are considered with respect to the same basic semantics.

Any laws of negation that directly follow from this core semantics will be taken to be the basic laws of negation, such that, if some "negation" operator does not satisfy them, it is not a *negation* operator. To consider such laws, we can define a notion of semantic entailment as follows:

> **Semantic Entailment:** $w \vDash \psi$ just in case, for any state w such that $w \Vdash \varphi$, $w \Vdash \psi$.

Simply in virtue of the core semantics for negation, the following entailment fact holds:

> **Minimal Contraposition:** If $\varphi \vDash \psi$, then $\neg\psi \vDash \neg\varphi$[6]

To see this, suppose $\varphi \vDash \psi$, so any state in which φ holds is a state in which ψ holds. Now suppose there is some state w such that $w \Vdash \neg\psi$, so there is no state compatible with w in which ψ holds. Since any state in which φ holds is a state in which ψ holds and there is no state compatible with w in which ψ holds, there is no state compatible with w in which φ holds. So $w \Vdash \neg\varphi$. Other than

[5] Though, it should be noted, that the paraconsistent logician might have some trouble living happily in this semantic framework.

[6] The name "minimal contraposition" (Berto 2015) is perhaps a bit misleading here, since this contraposition principle is definitive of *sub*minimal negation, with the definitive contraposition principle of minimal negation being the one that corresponds to the symmetry of incompatibility: $\varphi \vDash \neg\psi \Leftrightarrow \psi \vDash \neg\varphi$ (See Dunn 1999, 32).

minimal contraposition, what else follows from the very set-up? Berto claims that the symmetry of incompatibility, though it does not follow directly from the formalism itself like minimal contraposition, should be taken to be built into the core set-up.

To treat incompatibility as a symmetric notion is to impose symmetry on the accessibility relation C, such that, if v stands in C to w, then w stands in C to v. If we do *not* impose symmetry on the accessibility relation, a different notion of negation can be defined, for which Berto uses the symbol "$\sim$" rather than "$\neg$:"

S$_\sim$: $w \Vdash \sim \varphi$ just in case $\forall v(vCw \rightarrow v \nVdash \varphi)$

So, to take the same example, "$\sim$(a is blue)" holds in a state consisting in a's being red, since every state that is compatible with this state—for instance, the state consisting in b's being blue, the state consisting in a's being red and c's being yellow, and so on—is a state in which "a is blue" fails to hold. It's hard to resist the conclusion that "$\neg$(a is blue)" and "$\sim$(a is blue)" mean the very same thing: the thing would be expressed in English as "It's not the case that a is blue." If compatibility is symmetric, of course, they do mean the same thing; they hold in just the same states.

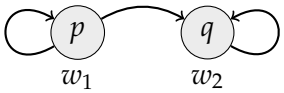
If we impose symmetry, such that "$\neg$ "and "$\sim$" express the same operation (namely, negation), that double negation introduction must hold. To see this, note first that $\neg$ and $\sim$ are related in the following way:

Galois Connection: $\varphi \vDash \neg\psi$ just in case $\psi \vDash \sim \varphi$

Now, from the trivial entailments, $\neg\varphi \vDash \neg\varphi$ and $\sim \varphi \vDash \sim \varphi$, we have the following entailments:

DNI$_A$**:** $\varphi \vDash \sim \neg\varphi$
DNI$_B$**:** $\varphi \vDash \neg \sim \varphi$

So, since, if we have symmetry, such that "¬ "and "∼" express the same operation, we have double negation introduction. Alternately, if we *don't* have symmetry, we *don't* have double negation introduction. To see this, consider the following set of states, with the arrows showing relations of compatibility between them:



Both p and q are compatible with themselves, p is compatible with q, but q is not compatible with p. To see that $p \vDash (\neg(\neg p))$ is invalidated by this model, first, see that p holds in w_1. Now, see that $\neg p$ holds in w_2, since w_2 is compatible with no states in which p holds. Finally, see that $\neg(\neg p)$ does not hold in w_1, since $\neg(\neg p)$ holds in w_1 just in case w_1 is compatible with no state in which $\neg p$ holds, but w_1 *is* compatible with a state in which $\neg p$ holds, namely w_2. Thus, in this model: $p \nvDash \neg(\neg p)$. If incompatibility must be symmetric, and the core meaning of negation is defined on the basis of incompatibility in this way, then any negation operator, as such, must satisfy double negation introduction.

2 The Question of Symmetry

But *must* (in)compatibility be symmetric? Could there be contents that stand in asymmetric (in)compatibility relations? Berto claims that there cannot be:

> Now (in)compatibility must be symmetric: whatever ontological kinds a and b belong to, it appears that if a rules out b, then b has to rule out a; that if a's obtaining is incompatible with b's obtaining, then b's obtaining must also be incompatible with a's obtaining; etc (2015, 779)

Now, presumably Berto is not really attempting to give an argument here; he's

simply appealing to our supposed intuition that incomaptibility must be symmetric. Still, given what he says, he might be construed as arguing inductively along the following lines. Pick some a and some b, be they properties, states of affairs, propositions, event-types, or what have you, such that a rules out b. Now confirm that b also rules out a. Repeat, picking a and b from different ontological categories until you are satisfied that (in)compatibility, in general, must be symmetric.

It turns out, however, that not everyone has found it to be clear that, for any a and b you pick, if a rules out b, then b rules out a; counter-examples have been proposed. For instance, if we think of *prevention* as a kind of incompatibility, such that a's preventing b is a way for a to be incompatible with b, then we get counter-examples.[7] Dunn (1996) gives the following example: Jon's practicing his saxophone prevents his father from reading a technical paper, but Jon's father's reading a technical paper does not prevent Jon from practicing his saxophone, (13-14). Now, Berto (2015) and then again Berto and Restall (2019) in response to De and Omori (2018) correctly, I think, diagnose the example as hinging on a conflation of the asymmetric causal relation of prevention and the symmetric non-causal relation of incompatibility. We must be able to tease these apart because a fact consisting in the former relation obtaining between two event-types might be explained by a fact consisting the latter relation obtaining between those two event-types. That is, an event-type a might be incompatible with an event-type b *because* the occurrence of a prevents the occurrence of b. If the fact that the occurrence of a prevents the occurrence of b *explains* the fact that a is incompatible with b, then the former fact cannot be *identical*

[7] See Hartonas and Dunn (1993, 14-15).

to the latter fact.[8] And it seems that we have this explanatory asymmetry in the proposed counter-example. Jon's practicing the saxophone is incompatible with his father's reading the paper *because* the occurrence of the first event-type prevents the occurrence of the second event-type. It follows from this that the prevention relation here is not identical to the incompatibility relation, and this allows us to say that it is only the prevention relation here (which explains the obtaining of the incompatibility relation) that is asymmetric; the incompatibility relation that obtains between these two event-types (which is explained by the prevention relation's obtaining between them) is symmetric.

Still, even supposing that this response to the proposed counter-example is successful, nothing about it is sufficiently general to ensure that other counterexamples won't arise. Here's a case that does not (at least seem to) hinge on asymmetric causal relations. Most people that I've asked seem to think that one person a can be romantically compatible with another person b, yet b can be romantically incompatible with a. Here, the ontological kinds are people, and the (in)compatibility relations are interpersonal ones between people. What should we say here?[9] In response to the saxophone example above, Berto says "Considerations involving asymmetrical causal relations should not sneak into the purity of our intuitions on the symmetry of (in)compatibility," (780). But this example seems to cast doubt on how pure our intuitions really are. Saying anything in response to this example requires us to say what the relevant notion of incompatibility is such that either people are not the right sort of relata or

[8]This is an instance of the Euthyprho schema: If someone's being pious explains their being loved by the gods, then their being pious cannot be identical to their being loved by the gods. Schematically, if some fact A explains some fact B, then A cannot be identical to B.

[9]Is this not a genuine case? Is romantic compatibility not the sort of compatibility at issue here? If so, what rules it out? Or are most people's intuitions wrong, and romantic compatibility, in fact, must be symmetric? Does defending a claim in the philosophy of logic really require us to venture into the philosophy of love?

that romantic (in)compatibility is not the right sort of relation. Now, the basic problem here, I take it, is that the relevant concept of (in)compatibility is the concept of a relation that obtains between *contents*, and people are not contents, so though there may be some sort of (in)compatibility relation that obtains between people that is somehow *analogous* to the relevant (in)compatibility relation that obtains between contents, it is not the very same relation. But the notion of "content" here—where concepts, properties, propositions, states of affairs, etc. count as contents, but people do not—is *not* something on which we have a clear intuitive grip. It is a philosopher's term of art. So, it seems that we cannot simply leave our intuitions untheorized and rely on the "purity of our intuitions on the symmetry of (in)compatibility."

To get to the real point here, even if we *do* have something to say about all of the cases that have been brought up that allows us to maintain that incompatibility is, in each genuine case, symmetric, it seems that *still*, all we can really conclude here is that it *appears* that incompatibility must be symmetric; we are not entitled to conclude that incompatibility, as such, *really must* be symmetric.[10] Entitlement to that latter claim requires one to be able to say *why* incompatibility must be symmetric, and neither Berto nor anyone else, as far as I'm aware, says anything to answer this question. Restall's (1999) langauge is much less committal with respect to the question of whether there could be asymmetric

[10]There is a whole class of potential counter-examples that I am not considering here of concern in the natural language semantics for conditionals and epistemic modals, in which the incompatibility relation defined by semantic theories actually is asymmetric, and that is in dynamic semantic theories meant to accommodate data like Reverse Sobel Sequences for conditionals or data involving epistemic "might." For conditionals, see, for insance, von Fintel (2001), Gillies (2007), Willer (2017) and, for epistemic modals see especially Lennertz (2018). I don't consider these kinds of cases here because the semantic *values* defined in these sorts of dynamic theories of that support asymmetric incompatibility relations can't be identified with semantic *contents*, as we one does in a standard truth-conditional semantics. This gets back to the point mentioned above, that we need to be clear about just what the relata of the relevant incompatibility relation actually are.

incompatibility relations. Laying out the same basic semantics, Restall says,

> We can consider some properties which it would be plausible to assume that C has. For example, compatibility certainly does *seem* to be symmetric. That is, if xCy then yCx, (1999, 62).

Here, Restall is more or less explicit that he is laying down symmetry as a constraint on the accessibility relation as a plausible assumption for doing formal semantics. Insofar as this is what one is doing, there is no problem here. However, there *is* a problem for what I have been calling "pros hen pluralism about negation," giving an account of the core meaning of negation and saying what facts about a negation operator must hold if that operator is to be properly called a negation operator according to that account. Berto purports to be giving an account according to which "nothing can be called a negation properly if it does not satisfy (Minimal) Contraposition and Double Negation Introduction," (761). As we've seen, double negation introduction holds just in case symmetry is imposed as a constraint on the accessibility relation C. So, if one purports to be giving an account of negation in terms of incompatibility, and one holds that it is an essential feature of something's genuinely counting as a negation operator that double negation introduction holds, then one had better be able to say why incompatibility must be symmetric. But neither Berto, Restall, nor anyone else who has proposed this incompatibility-based account of negation, has anything to say here.

It is worth being explicit that, whatever the problems with the classical conception are, the classical logician clearly does not have this problem.[11] It is clear, on the classical conception, what the laws of negation are, double negation

[11]Nor, we should note, does the non-classical logician who follows the "American plan," thinking of negation as a contradictory forming operator as the classical logician does, but distinguishing untruth from falsity so as to maintain that a sentence might be neither true nor false or both true and false. See De and Omori (2018, 292-296) on this point.

introduction being one of them. p entails $\neg(\neg p)$ because, if p is true, then $\neg p$ is false, and if $\neg p$ is false, then $\neg(\neg p)$ is true. This fact holds in virtue of the account of the basic semantic function of the negation operator. On the classical conception, what the negation operator does, when prefixed to a sentence that is either true or false, is form a sentence that has the opposite truth value, *false* if the original sentence is true, *true* if the original sentence is false. Double negation introduction follows directly from this conception of what a negation operator is and does. Of course, lots of other laws also follow it, for instance, the much more contentious double negation elimination, a "law" that our constructive logician rejects. However, if one wants to employ a weaker logic, one in which some of these classical "laws" of negation are not laws, one has to be able to give an account of the negation operator that one is using such that it is clear, according to that account, what the laws of negation are. Berto claims to be able to do this. However, insofar as he has nothing to say as to why it is that incompatibility must be symmetric, he is not entitled to this claim.

Now, at this point, Berto can retreat and revoke his commitment to the claim that he has really offered an account of negation according to which nothing can be called a negation properly if it does not satisfy double negation introduction. So-called "sub-minimal" negations have been explored which do not validate double negation introduction (Dunn 1993, Hazen 1995). It seems to me at least, however, that a minimal negation operator really is the *minimal* negation operator, in the sense that nothing weaker than it really is a negation operator. Here's an argument for this claim: If p, then clearly not *not p*. After all, $\boldsymbol{p}$. So, not *not p*. If one gives an account of the propositional operator expressed by "¬," such that this argument is no good when one substitutes one's "¬" for "not," then, whatever one's "¬" means, it doesn't mean *not*. If we want to vindicate

the incompatibility-based account of the core meaning of negation, we must be able to say why incompatibility must be symmetric.

3 The Normative Pragmatic Conception

In explicating the concept of material incompatibility or exclusion, Berto (2008) makes it clear that the notion is to be understood *semantically*, in terms of *contents*, rather than *pragmatically*, in terms of *acts*. He tells us:

> Put it any way you like, material exclusion has to do with *content*, not mere performance: it is rooted in our experience of the world, rather than in pragmatics, (Berto 2008, 180).

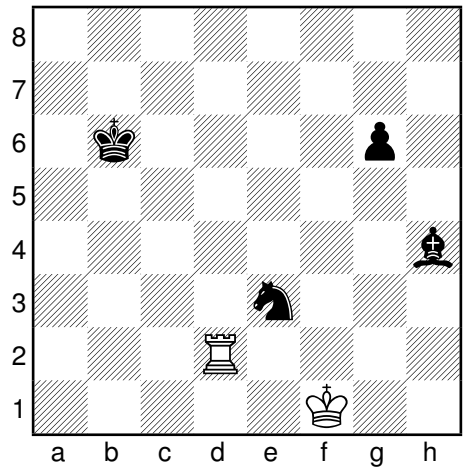
I have followed this lead above, thinking of the relata of the incompatibility relation as states of affairs, presumably, consisting in objects having properties and standing in relations. From this perspective, the question of why incompatibility must be symmetric seems to me to be pretty intractable. It seems to require saying something in general about what it is for something to be a state of affairs, such that, saying this, we are able to say that, for any two possible states of affairs a and b, if a is incompatible with b, then b is incompatible with a. If we continue to think in these terms, I do not know what lines of thought about the nature of states of affairs could entitle us to this conclusion. In order to make progress on the question of the symmetry, I think that this assumption that Berto makes here is precisely the one that we must reject.

Against Berto, I suggest that we don't think about incompatibility, in the first instance, as an *alethic* modal relation between *contents*, such as states of affairs, but, rather, as a *deontic* modal relation between *acts*, specifically, acts of *making claims*.[12] Now, of course, acts of making claims have contents—in making a claim

[12]It's worth noting that, in other work regarding the "bilateral" reading of multiple conclusion

one expresses a proposition—and such a proposition will be true just in case a certain state of affairs obtains. However, the thought I want to pursue here, owed to Brandom (and his Hegel) (2008, 2019), is that we can only understand the *alethic* incompatibility relations that obtain between these states of affairs by thinking through the *deontic* incompatibility relations that obtain between the claims that are made true by the obtaining of these states of affairs. I am proposing, then, that we think about incompatibility, in the first instance, not in terms of what it is for two *worldly entities* to be *alethically* incompatible, but it terms of what it is for two *discursive acts* to be *deontically*, or, as I will say from hereon out, *normatively* incompatible.

To get into the frame of mind that I am encouraging us to adopt, let me start with an analogy. Consider the position occupied by the white player in this game of chess, with white to play:



In this position, since one's king is in check, one is *compelled*, given the rules of the game, to move one's king to a different position, and being so compelled

sequent calculi, Restall (2005) adopts a normative conception of incompatibility quite like the one I am about to suggest. See Hlobil (M.S.) for a complementary proposal relating Restall normative bilateralism to truth-maker semantics, articulated in alethic modal terms.

precludes one from being able to enter into other positions in the game that one would otherwise be able to enter into. For instance, one's king's being at f1 precludes one from being able to enter into the position that one would occupy by moving one's rook to d6, a move that, were one's king not at f1, one would be able to to make. The agentive modal operators "compelled" and "able" here are to be heard in *normative* register.[13] One is precluded from being able to move one's rook to d6 not in the sense that one is physically incapable of doing so, but in the sense that doing this is not a legal move in the game. It is, in that sense, a move that is precluded, excluded, or ruled out.

The analogy comes from thinking of acts of making claims as moves in what Brandom (1994) calls "the game of giving and asking for reasons." In the particular incarnation of the game of giving and asking for reasons that speakers of English play, when one occupies the position that one comes to occupy by saying "*a* is red," one is precluded from being able to occupy the position that one comes to occupy by saying "This is blue." Here again, the agentive modal operator "able" is to be heard in *normative* register. When one has made the claim "*a* is red" (or has made some other set of claims that forces one into the position one occupies by making that claim), one is precluded from being able to make the claim "This is blue," not in the sense that one is physically incapable of doing so, but in the sense that, once one makes the claim "*a* is red," making the claim "*a* is blue," is not a legal move. To make this clear, we might say that one is precluded from being *entitled* to make the claim "*a* is blue," given that one is *committed* to the claim "*a* is red." This is Brandom's normative pragmatic characterization of what it is to take two claims to be incompatible:

> In practical terms of normative status, to treat p and q as incompatible

[13] See Mandelkern, Schultheis, and Boylan (2017) for a discussion of this flavor of modality.

> claims is to take it that commitment to one precludes entitlement to the other, (1994, 115).

So, to take the claims "*a* is red" and "*a* is blue " to be incompatible is to take *commitment* to "*a* is red" to preclude *entitlement* to "*a* is blue" and vice versa. The "vice versa" is, of course, what we're concerned with here. As Brandom defines it, incompatibility is symmetric: two claims are incompatible just in case commitment to *one* precludes entitlement to *the other*. But here too, this way of thinking about incompatibility does not itself mandate symmetry; the logically prior concept with which Brandom is working is what I'll call "preclusive consequence," where p stands in a relation of preclusive consequence to q just in case commitment to p precludes entitlement to q. Incompatibility is defined as preclusive consequence in both directions, but the concept of preclusive consequence in terms of which incompatibility is defined does not, as such, mandate symmetry. The question, translated into this idiom, is: must preclusive consequence be symmetric? Is it possible for there to be two claims, p and q, such that commitment to p precludes entitlement to q but not vice versa? Brandom, like Berto, says nothing to answer this question, simply assuming without argument that preclusive consequence is symmetric. However, this question, I now hope to show, is much more tractable.

The first step to answering this question is to answer why there must be preclusive consequence relations in the game of giving and asking for reasons at all. What is the role that these consequence relations play in the game of giving and asking for reasons? Before we consider whether the game could be played with these relations being asymmetric, let us consider whether the game could be played without them at all. I submit that it could not be. Without preclusive consequence relations, making a move could not be counted as *challenging*

another move. To make a move is to undertake a *commitment* to demonstrate one's *entitlement* to that move. Without challenges in the practice, which compel one to demonstrate one's entitlement to a move, the very idea that what one is doing in making a move is undertaking a commitment is lost. Challenges are, in a sense, the keystone that holds together the structure of the game of giving and asking for reasons. If there is no giving of reasons, there is no reasoning, and there is no giving of reasons if there is no calling for them.

What is the most basic case of a challenge? A challenger, in a way that is directed at the challengee, makes a claim that is incompatible with the one that they intend to challenge.[14] Where incompatibility is understood as an invertable relation of preclusive consequence, this makes sense. If A makes the claim q, and B makes the claim p, where it is mutually acknowledged that commitment to p precludes entitlement to q and commitment to p precludes entitlement to q, it is mutually acknowledged by A and B that the commitments they have respectively undertaken are normatively incompossible: they cannot both be taken on. Insofar as making a claim paradigmatically does not only aim to *entitle* other players to it but also puts a demand upon them to *commit* themselves to it, A's making q and B's making p leads to a situation in which it is mutually acknowledged that A must show that their claim to being entitled to q is stronger than the B's claim to being entitled to p, or else A must revoke their commitment to q, and likewise for B. But what happens if A takes commitment to p to preclude entitlement to q but not vice versa? Well, let's see.

[14] See Wanderer (2010) on the way in which a challenge must be second-personally directed.

4 A Dialogue

A: q

B: So, you are committed to q?

A: Yup.

B: Well, I challenge this commitment of yours on the following grounds: p, p is incompatible with q, and my grounds for p are better than your grounds for q.

A: I agree that p is incompatible with q, and you're grounds for p are better than my grounds for q.

B: Surely, then, you must revoke your commitment to q.

A: No. I will, however, commit myself to p now, since you've given good grounds for p.

B: How can you take yourself to be able to do that? You just agreed with me that p is incompatible with q, so, insofar as you are committed to q you are precluded from being entitled to p. So you *can't* commit yourself to p.

A: That is a complete non-sequitur. Commitment to p is incompatible with q, so, commitment to p precludes entitlement to q, but q isn't incompatible with p, so commitment to q does not preclude entitlement to p. Accordingly, I can commit myself to p, and that's what I just did.

B: No! You *can't* do that.

A: Why not?

B: You grant that commitment to p precludes entitlement to q, right?

A: Right.

B And you're committed to both p and q, right?

A: Right.

B: So, you're both committed and precluded from being entitled to q! You have an incoherent set of commitments, and you have an incoherent set of commitments in virtue of commitment to p and q. Now, you've already acknowledged that the grounds for p are much better than the grounds for q. Accordingly, since you must rectify your incoherent set of commitments, you must revoke your commitment to q.

> *A*: Once again, that is a complete non-sequitur. Although commitment to p *alone* precludes entitlement to q, commitment to q and p *together* does not preclude entitlement to q. So, since I was committed to q already, and commitment to q does not preclude entitlement to p, I could commit myself to p, without precluding myself from being entitled to q, and that's just what I've done. Once I did this, of course, I was committed to p, but, because commitment to q and p together does not preclude entitlement to q, I am not precluded from being entitled to q.
>
> *B*: Let me get this straight: *You* can be committed to q while also being committed to p because you're committed to p and q, and commitment to p and q together does not preclude entitlement to q, but *I* can't be committed to q, because I'm precluded from being entitled to it, since I'm committed to p without being committed to q, and commitment to p precludes entitlement to q.
>
> *A*: Exactly.

The first part of this dialogue brings out that, once we think of incompatibility in terms of its role in challenging claims, the possibility of an asymmetric incompatibility relation, where commitment to p precludes entitlement to q but commitment to q does not preclude entitlement to p, leads to a very strange dialogical situation. Insofar as A is committed to q, and does not take commitment to q to preclude entitlement to p, they must take themself to be able to take on a commitment to p, even given a commitment to q.[15] After all, if their commitment to q rules out their being able to take on a commitment to p, then another way to say that very thing would be to say that their commitment to q precludes them from being entitled to take on a commitment to p, and that is just what we are supposing is not so. So, if A takes commitment to p to preclude entitlement to q, and so takes it that, given a commitment to q, they can commit themself to p,

[15] I use "themself" here, since A is a singular discursive agent of unspecified gender. It's relevant to the point here that A could be, for instance, an extra-terrestrial from a culture with five genders or an artificial intelligence from a future society with no genders. The most widely-accepted singular personal genderless pronoun in English is "they" and its reflexive form, when used as a singular rather than plural pronoun, is "themself."

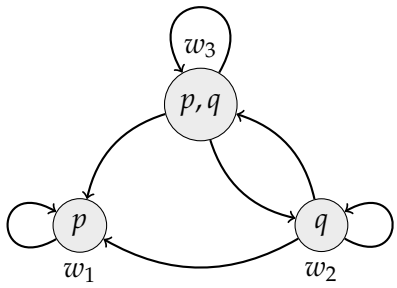
they must also take it that, though commitment to p alone precludes entitlement to q, commitment to p and q together does not. I take it that this is probably sufficiently strange for most people to conclude that, if incompatibility is understood in terms of its role in challenging claims, it must be symmetric. But, though there's clearly something wrong here, it is not immediately clear what, in particular, it is.

The first thing to note is that A's attitude here involves a rejection of the *monotonicity* of preclusive consequence, the schema that, for any claims φ, ψ, and χ, if commitment to φ precludes entitlement to ψ, then commitment to φ and χ together must still preclude entitlement to ψ. Now, in the formal framework spelled out here, given the reflexivity of the inclusion relation, it follows from (Backwards) that one cannot make two incompatible states compatible by adding information to one of them. Conceptualizing this fact in the current normative rendering of "states," this comes out to the claim one can't make two incompatible states of commitments compatible by adding an additional commitment to one of them. Since A must suppose this is possible in taking the incompatibility relation between p and q to be asymmetric, one might think this suffices There is reason to think, however, this is simply a technical shortcoming of that formal framework that we've laid out, and it cannot be appealed to in order to make a conceptual point about the notion of incompatibility on which the framework is based. Incompatibility is *not*, in general, monotonic.[16] In making this point, it's important to be clear once again that the primary notion

[16]Depending on one's purposes in adopting a formal framework, idealizing away from this fact may or may not be acceptable. For non-monotonic formal semantic frameworks, in which negation is still understood in terms of (material) incompatibility, see, for instance, the phase space semantics proposed by Porello (2012) or Kaplan (2021), or the truth-maker semantics proposed by Hlobil (M.S.). It's worth noting that these frameworks all suppose that incompatibility is symmetric so the argument I am providing here can be exported from this formal context to any of those.

of incompatibility that we're dealing with here is *material* incompatibility, not formal incompatibility, and, for most examples of material incompatibility that we're naturally inclined to think up, we can find defeaters. For instance, to take an example considered above, Jon's practicing the saxophone is incompatible with his father's reading a technical paper, but his practicing the saxophone and doing so with a mute in it is not incompatible with his father's reading a technical paper.[17] So, to get back to the dialogue, what's wrong A's position here can't simply be their taking it that commitment some claim φ precludes entitlement to ψ, but that commitment to φ along ψ doesn't.

If it's not the general schema of monotonicity that's wrong with A's attitude, it must be something related to more specific schema that if commitment to φ precludes entitlement to ψ, then commitment to ψ and φ together must still preclude entitlement to ψ. Once again, it is certainly very *strange*, but it's not immediately clear that there's a non-question-begging argument for the claim that it is *incoherent* to take commitment to φ to preclude entitlement to ψ but to also take commitment to ψ and φ together to not preclude entitlement to ψ. To see how things look, from A's perspective, it might be helpful to draw a picture:



[17] It's worth noting that every participant to the debate has been happy to countenance this example as a genuine case of incompatibility of the relevant sort. Of course, one might retract this acceptance, but, if you rule out all such examples, you're going to end up saying no claims, except, perhaps claims about the colors and shapes of monochromatic solids, are really incompatible.

Here, we have the three states under discussion here: the state of being committed to p, the state of being committed to q, and the state of being committed to both p and q. The arrows show relations of compatibility between these states. A state w is *compatible* with a state v just in case occupying w, having all the commitments had by one who occupies w, *leaves open* entitlement to all the commitments had by one who occupies v. A state w is *incompatible* with a state v just in case occupying w, having all the commitments had by one who occupies w, *precludes* entitlement some of the commitments had by one who occupies v. A and B agree that p is incompatible with q. That is, the state of being committed to p does not leave open, but, rather, precludes entitlement to q. That is why there is no arrow going from the w_1 to w_2. However, A claims that q is compatible with p. That is, the state of being committed to q leaves open entitlement to p. That is why there is an arrow going from the w_2 to w_1. Furthermore, A claims that, while p is incompatible with $\{p, q\}$, q is compatible with $\{p, q\}$. So, there is no arrow going from w_1 to w_3, but there is an arrow going from w_2 to w_3. Finally, no state here is, as such, incoherent. That is, no state is such that occupying it commits one to a claim to which one is precluded from being entitled. So, every state has an arrow going to itself. Now, perhaps this is an unusual set of relations between states, but, once again, it's not immediately clear why it's incoherent or absurd for A to think that these three states stand in these relations.

To see what is ultimately wrong with A's attitude, we must see what B is able to do, given that A has this attitude. Let us continue the dialogue:

> B: Ok, well then I revoke my commitment to p.
>
> A: Why would you do that? You have good reasons for p.
>
> B: Sure, I have good reasons, but certainly it must be permissible, in response to a challenge to revoke one's commitment to the challenged claim. A challenge essentially presents me with a *choice*: to either give

reasons for my challenged claim or to revoke my commitment to it. I opt for the latter.

A: Alright, sure, that's indeed something you can do.

B: Ok, so I revoke my commitment to p. I now accept q.

A: You accept q rather than p? But your reasons for p are *better* than your reasons for q.

B: Well, that's no problem now that I've revoked my commitment to p and accepted q. Reasons *for* p are only reasons *against* q insofar as I'm committed to p. Now that I've revoked my commitment to p and accepted q, reasons for q are not reasons against p. Since there are good reasons for p, I'll now accept p again. We're in the same state now.

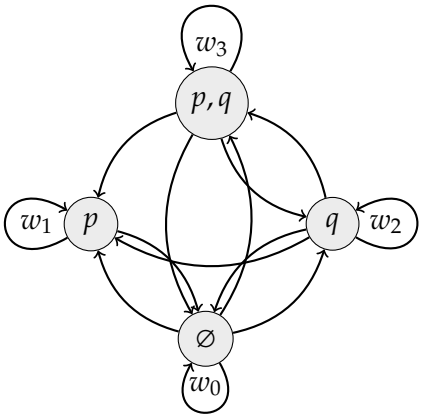
A: Hmm . . . I guess so. Alright, great. I'm glad we were able to settle our disagreement.

B: No! You never *acknowledged* our disagreement! *I* challenged your claim to q on the grounds that p, p is incompatible with q, and the grounds I have for p are better than the grounds you have for q. *You* claimed to agree with all of this. You said that you took p to be incompatible with q, and you agreed that my grounds for p were better than your grounds for q. You insisted, however, that, though p was incompatible with q, q was not incompatible with p. But you never *really* took p to be incompatible with q at all.

A: Sure I did. Isn't that why we had this whole discussion?

B: No, you never really took commitment to p to preclude entitlement to q. The whole time, you took it that someone who is committed to p *can* take on an additional commitment to q. You just took it that, in order to do this, they have to go through the detour of giving up p, accepting q, and then accepting p again. But this detour that you take it that someone who is committed to p must take in order to be committed to both p and q is *pointless*. It doesn't require one to take on any additional commitments. So someone who is in a state of being committed to p can, without taking on any commitment other than q, enter into a state of being committed to both p and q. From a pragmatic perspective, your attitudes are exactly the same as those of someone who allows one who is committed to p to take on a commitment to q directly. So, I repeat, you don't *really* take p to be incompatible with q. Really taking p to be incompatible with q requires also taking q to be incompatible with p.

B reveals that the set of states at play here is really of the following form:



As this picture shows, someone in w_1, who is committed to p, *can* get to w_3, taking on an additional commitment to q, simply by rejuggling their commitments. What someone in w_1 ought to do in order to get to w_3 is revoke their acceptance of p, which takes them back to w_0, then accept q, taking them to w_2, and then accept p, taking them to w_3. As we've said above, it follows from the minimal constraint on the compatibility relation that the null-state is compatible with every state that is compatible with some state. Since every state we're considering here is compatible with itself, the null-state is compatible with every state. So it is always licit to go to the null-state in order to get to "the other side" of the incompatibility, and proceed from there.

The claim that B makes, concluding the above dialogue, is that asymmetric incompatibility is no real incompatibility at all. Incompatibility, understood pragmatically as I am understanding it here, must, at least potentially, be able to bear in *challenging* a claim. If p is incompatible with q, in the sense that commitment to p precludes entitlement to q, then an act of claiming q must be able to function to challenge to someone's commitment to p, even if we suppose

that the converse does not hold. However, in this case, A claims to take p to be incompatible with q, and yet, A's claim of q is *not* able to function to challenge to B's commitment to p, since B can simply rejuggle their commitments and, maintaining a commitment to p, commit themself to q in a way that is perfectly licit by A's lights. In this way, A's attitude of taking p to be incompatible with q is utterly pragmatically inefficacious, and, for this reason, B claims that A does not really have this attitude at all. We've considered the simplest case here, involving only two asymmetrically incompatible claims, but the point here extends straightforwardly to cases of more than two claims: asymmetric incompatibility is no incompatibility at all.

This argument bears a certain resemblance, at a high enough level of abstraction, to the Dutch Book arguments in probability theory. These arguments show that, if a practice of betting and bet-taking doesn't correspond to the probability axioms, then it will be defective in such a way that one player can essentially cheat the other out of all of their money, no matter what actually ends up happening with the events the bets are being placed on. This is a certain kind of "pragmatics-first" justification of the formal structure of the probability calculus, explaining why the calculus must have the structure that it does by showing that any deviation from this structure would correspond to a defective betting practice, where one player is able to cheat the other out of all of their money no matter what, effectively trivializing the whole practice of betting. Likewise, we've offered a pragmatics-first justification of the formal structure of a certain semantic relation, incompatibility, explaining why it must have the symmetric structure that it does since a deviation from it would correspond to a defective practice of challenging, one in which a player is able to essentially "cheat" themselves out of a challenge by simply rejuggling their commitments. The whole

point of challenging is that it's supposed to make someone *choose*—to either give up the challenged commitment or defend it. If one doesn't take incompatibility relations to be symmetric, there's a way to "cheaply win" the game of giving and asking for reasons, no matter what one's actual reasons are, trivializing the whole practice.

5 Conclusion: A Semantic One from Pragmatic Premises

I started this paper with an example involving worldly states of affairs: a car's being red is incompatible with its being blue. These worldly states of affairs might be identified as the *semantic contents* of the sentences "This car is red" and "This car is blue."[18] On the account of negation proposed by Berto, we are to understand incompatibility as a semantic relation between contents. I have claimed, by contrast, that only if we understand it, in the first instance, as a pragmatic relation between acts can explain why it is that it must be a symmetric relation. Still, the following question may seem to linger: how does this get us back to our original explanandum? How have we thereby explained the symmetry of the semantic relation between the contents, rather than merely the pragmatic relation between the acts? To answer this question, it is necessary to contextualize this approach in the pragmatist theory of content put forward by Brandom, according to which the contents of claims are *conferred* by underlying practice of making claims and giving and asking for reasons for them. If we think of contents as conferred by practices in the way suggested by Brandom, then we can account for general structural features of contents by

[18]One might alternately think of the semantic contents of these sentences as the propositions made true by the obtaining of these states of affairs—this distinction doesn't matter for the present point.

explicating the necessary structural features of any practice capable of conferring contents. This not only enables us to make sense of the argument here, but enables us to situate it in a wider class of "pragmatic transcendental arguments," which can function to provide explanations of structural features of contents or concepts by explicating them in terms of corresponding structural features of any possible discursive practice. In this connection, let me close by connecting the transcendental argument I've given here, drawing on Brandom, with another transcendental argument that has actually been made by Brandom.

Let us turn our attention, for a moment, from the symmetry of incompatibility to the symmetry of identity. However absurd it might have seemed to think (at least at the start of this paper) that one could offer an argument for the symmetry of incompatibility, it might seem monumentally more absurd to think that one could offer an argument for the symmetry of identity. In effect, however, that is just what Brandom does in Chapter Six of *Making It Explicit*, though he never quite puts it that way. Brandom identifies singular terms as the types of expressions that can be substituted *for* one another in different sentence frames. Accordingly, an identity claim such as "$a = b$" expresses the goodness of a substitution inference from a claim of the form Fa (for any sentence frame Fx) to a claim of the form Fb. The question of the symmetry of identity, then, becomes the question of whether these sorts of substitution inferences must be symmetric, or whether is it possible for there to be a discursive practice in which they are asymmetric? Brandom purports to show that no discursive practice that is capable of introducing conditionals could support expressions that are asymmetrically substitutable into sentence frames. Thus, insofar as a discursive practice is such that reasons can be made explicit, in the form of conditionals, the concept of identity on which members of that practice have a grip must be

symmetric. I will not rehearse the details of Brandom's argument here (see 1994, 376-384). The point is just to note the structure of the answer to the question of the symmetry of identity that Brandom's argument can be seen as providing. The notion of identity is understood, in the first instance, *pragmatically*, in terms of what one *does* when one makes an identity claim—what kind of inferences one licenses. Brandom then shows that any practice in which identity claims function in this way could not actually function as a full-blown discursive practice.

This all becomes relevant to the present discussion when we consider how Berto takes incompatibility to be a *primitive* notion, and presumably takes this fact to mean that there is no way to give an account of its basic features such as its symmetry. Berto and Restall (2019) double down on this point, claiming that incompatibility belongs to a class of other "fundamental notions like reference, identity, necessity, or negation," (2019, 4). The basic structural features of these notions, it seems, can only be *explicated*; they can't be *explained*. As Berto says, "Explanations stop when we reach concepts that cannot be defined in terms of other concepts, but only illustrated by way of example," (2015, 768). But there is another kind of explanation of a concept that is distinct from a semantic *re*duction of that concept, and that is a pragmatic *de*duction of that concept. I hope to have shown, by way of example, what such a pragmatic deduction can look like.[19]

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