Simply Substructural Semantics

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The Plan

- Establish some intitial common ground about the aim and desiderata of a semantifc theory.
- Explicate the dominant truth-conditional approach in semantics and its account of semantic implication and incompatibility relations.
- Raise a problem for this approach concerning relations of implication and incompatibility that don't conform to the structural principles such a semantics imposes.
- Spell out a radical alternative: an inferentialist approach to semantics that is substructural from the outset.
- Develop some general considerations for why this approach ought to be taken seriously.

Common Ground
The Aim of Semantics

- **A Simple Example:** Consider the fact that competent speakers English never utter “The ball is red” and “The ball is green” at the same time.
  - This is a bit of behavior that is presumably explained by their knowledge of the meanings of these sentences.
- **Yalcin:** “I take it that in natural language semantics, the aspect of reality we are seeking some understanding of is a dimension of human linguistic competence—informally, knowledge of meaning. Competent speakers of a language know (‘cognize’, etc.) the meaning features of expressions of their language. The semanticist is interested in modeling this state of mind and the associated semantic features,” (2018, 353).
- We do this by assigning *semantic values* to expressions.
  - Mathematically defined models of meanings.
  - Explain semantic competence by modeling knowledge of meaning as knolwedge of semantic values.

Common Ground

One Key Function of Semantic Values

- **Yalcin Again:** "[S]emantic values are assumed to be the sorts of things consequence and consistency relations are articulated in terms of: when $\Gamma \vDash \varphi$ holds, this is (at least partly) because of the semantic values of (the sentences in) Γ and of φ, respectively. Hypotheses about semantic values can thereby serve to predict, and ground, entailment and consistency facts, hence knowledge of such facts," (2014, 24).
- **Important Point:** Consequence and (in)consistency relations are *pre-theoretical* notions that a semantic theory aims to explain.
  - Since "entailment" is a theoretical notion, I'll use the more neutral term "implication" to express the pre-theoretical notion of one sentence's following from another.

The Truth-Conditional Picture
The Basic Framework

- **The Key Idea:** To know the meaning of a sentence φ is to know how the world would have to be, among all the ways it could possibly be, in order for φ to be true.
- **Implementating the Key Idea:** We start with a space of possible worlds W such that, for any $w \in W$ and any atomic sentence p, p is either true in w or false in w.
  - $[\![p]\!] = \{w : p \text{ is true in } w\}$
  - $[\![\neg\varphi]\!] = W - [\![\varphi]\!]$
  - $[\![\varphi \wedge \psi]\!] = [\![\varphi]\!] \cap [\![\psi]\!]$
  - $[\![\varphi \vee \psi]\!] = [\![\varphi]\!] \cup [\![\psi]\!]$
- **Implication:** $\varphi_1, \varphi_2 \ldots \varphi_n \vdash \psi$ just in case $([\![\varphi_1]\!] \cap [\![\varphi_2]\!] \ldots [\![\varphi_n]\!]) \subseteq [\![\psi]\!]$
- **Incompatibility:** $\varphi_1, \varphi_2 \ldots \varphi_n \perp \psi$ just in case $[\![\varphi_1]\!] \cap [\![\varphi_2]\!] \ldots [\![\varphi_n]\!] \cap [\![\psi]\!] = \varnothing$

The Truth-Conditional Picture
Some Good Results

- Good result one:
  - ✓ "The ball is red" implies "The ball is not green."
- Good result two:
  - ✓ "The ball is not colored" is incompatible with "The ball is red or blue."
- Good result three:
  - ✓ " The ball is a primary color, and the ball is not red or blue" implies "The ball is yellow."
- Good result four:
  - Every implication and incompatibility of classical logic.

The Truth-Conditional Picture

The Structure of Implication and Incompatibility

- **Monotonicity Principles:**
  - **MO1:** If $\varphi \vdash \psi$, then $\varphi, \chi \vdash \psi$
  - **MO2:** $\varphi \perp \psi$, then $\varphi, \chi \perp \psi$
- **Transitivity Principles:**
  - **T1:** If $\varphi \vdash \psi$ and $\psi \vdash \chi$, then $\varphi \vdash \chi$.
  - **T2:** If $\varphi \vdash \psi$ and $\psi \perp \chi$, then $\varphi \perp \chi$.
- All of these follow directly from our definitions of implication and incompatibility and the fact that semantic values are sets.

The Truth-Conditional Picture
Some More Good Results

- Good result one:
  - ✓ “The ball is crimson” implies “The ball is red.”
  - So, ✓ “The ball is crimson” along with “The ball is rubber” implies “The ball is red” (by MO1).
- Good result two:
  - ✓ “The ball is crimson” implies “The ball is red”
  - ✓ “The ball is red” implies “The ball is colored”
  - So, ✓ “The ball is crimson” implies “The ball is colored” (by T1)
- Good result three:
  - ✓ “The ball is crimson” implies “The ball is red”
  - ✓ “The ball is red” is incompatible with “The ball is green”
  - So, ✓ “The ball is crimson” is incompatible with “The ball is green” (by T2)

The Problem of Structure
Some Bad Results

- **The Basic Problem:** Intuitive relations of implication and incompatibility often don't conform the the structural principles just stated.
- Bad result one:
  - ✓ "The lawn is grass" implies "The lawn is green"
  - So, # "The lawn is grass" along with "The lawn is completely sun-scorched" implies "The lawn is green" (by MO1)
- Bad result two:
  - ✓ "Bella's a bird" implies "Bella flies"
  - ✓ "Bella's a penguin" implies "Bella's a bird"
  - So, # "Bella's a penguin" implies "Bella flies" (by T1)
- Bad result three:
  - ✓ "Sadie's a mammal" is incompatible with "Sadie lays eggs."
  - So, # "Sadie's a mammal" along with "Sadie's a platypus" is incompatible with "Sadie lays eggs" (by MO2)

The Problem of Structure

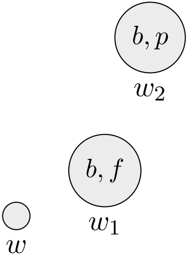
The Variably Strict Proposal

- **The Basic Idea (based on Lewis and Stalnaker on conditionals):** A sentence φ implies a sentence ψ in a context c not only if *all* of the φ-worlds are ψ-worlds, but, rather, if the *closest* φ-worlds are ψ-worlds, where closeness is contextually determined by one's expectations for things in the world:
  - **Closeness:** A world w is closer to the actual world than a world v, in a context c, $w \leq_c v$ just in case w is as much or more like the way one expects the world to be in c than v
  - **Minimally Distant φ-Worlds:** $\min_{\leq_c}(\llbracket\varphi\rrbracket) = \{w \mid w \in \llbracket\varphi\rrbracket$ and, for all v, if $v \in \llbracket\varphi\rrbracket$, then $w \leq_c v\}$
  - **(Potentially Defeasible) Implication:** $\varphi_1, \varphi_2 \ldots \varphi_n$ (perhaps defeasibly) implies ψ in c just in case $\min_{\leq_c}(\llbracket\varphi\rrbracket \cap \llbracket\varphi_2\rrbracket \ldots \cap \llbracket\varphi_n\rrbracket) \subseteq \llbracket\psi\rrbracket$
  - **(Potentially Defeasible) Incompatibility**: $\varphi_1, \varphi_2 \ldots \varphi_n$ are (perhaps defeasibly) incompatible with ψ in c in case $\min_{\leq_c}(\llbracket\varphi\rrbracket \cap \llbracket\varphi_2\rrbracket \ldots \cap \llbracket\varphi_n\rrbracket) \cap \llbracket\psi\rrbracket = \varnothing$

The Problem of Structure

The Variably Strict Proposal: Some Good Results

- MO1, MO1, T1, and T2 no longer imposed.
- Accommodates all the cases just considered.
- **Example of Bella:**



  - $b \vdash f$ but $b, p \nvdash f$
  - $p \vdash b$ and $b \vdash f$, but $p \nvdash f$

The Problem of Structure

The Failure of the Variably Strict Proposal

- ***Cumulative* Transitivity Principles:**
  - **CT1:** If $\varphi \vdash \psi$ and $\varphi, \psi \vdash \chi$, then $\varphi \vdash \chi$.
  - **CT2:** If $\varphi \vdash \psi$ and $\varphi, \psi \perp \chi$, then $\varphi \perp \chi$.
- These principles are generally thought to be unproblematic. *But they're not!*
- Bad result one:
  - ✓ "The lawn is grass" implies "The lawn is green"
  - ✓ "The is grass" along with "The lawn is green" implies "The lawn is not completely sun scorched."
  - So, # "The lawn is grass" implies "The lawn is not completely sun-scorched."
- Bad result two:
  - ✓ "Bella's a bird" implies "Bella flies"
  - ✓ "Bella's a bird" along with "Bella flies" is incomaptible with "Bella's a penguin"
  - So, # "Bella's a bird" is incompatible with "Bella's a penguin"

The Problem of Structure
Remaining Options

- Standard non-monotonic logics (e.g. Adam's (1975) probabilistic logic, Kraus, Lehmen, and Magidor's (1990) preferential logic, Horty's (2007, 2012) default logic) all impose Cumulative Transitivity.
- Dynamic proposals are possible (e.g. Simonelli M.S. following von Fintel (2001)), but none currently exist and to deal with the whole range of these cases
  - things get very complicated very quickly
  - things end up looking rather ad hoc
- I want to suggest the these issues are simply and elegantly resolved if we think of things in the opposite direction—starting with inferential relations rather than explaining them in terms of representational ones.

The Substructural Inferentialist Framework

Representationalism vs. Inferentialism

- **The Representationalist Order of Explanation:**
  - A sentence's meaning is understood in the first instance in terms of the way it represents the world as being (modeled as the set of possible worlds that are that way).
  - Inferential relations between sentences are understood in terms of the relations between their representational contents (modeled set-theoretically).
- **The Inferentialist Order of Explanation**
  - A sentence's meaning is understood, in the first instance, in terms of the inferential relations it bears to other sentences.
  - The representational content of a sentence—the proposition expressed by it or state of affairs on which its truth turns—is understood as a *reification* of its inferential role, (Sellars 1963, 1968).

The Substructural Inferentialist Framework

Normative Bilateral Inferentialism

- **Normative Conception of Semantic Relations:** Think of the relations of implication and incompatibility in *normative* terms, using the dual normative statuses of *commitment* and *entitlement* (Brandom, 1994).
- **Brandom on Incompatibility:** "In practical terms of normative status, to treat p and q as incompatible claims is to take it that commitment to one precludes entitlement to the other," (1994, 115).
- **Brandomian Bilateralism:** In the style of Rumfitt (2000), we'll use signed formulas to express "normative positions," writing
  - $\oplus\langle\varphi\rangle$ to express that φ is *included* in one's set of *commitments*.
  - $\ominus\langle\varphi\rangle$ to express that φ is *precluded* from one's set of *entitlements*.
- **Implication as Committive Consequence:** $\varphi_1, \varphi_2 \ldots \varphi_n$ implies ψ just in case $\oplus\langle\varphi_1\rangle, \oplus\langle\varphi_2\rangle, \ldots \oplus \langle\varphi_n\rangle \Rightarrow \oplus\langle\psi\rangle$
- **Incompatibility as Preclusive Consequence:** $\varphi_1, \varphi_2 \ldots \varphi_n$ is incompatible with ψ just in case $\oplus\langle\varphi_1\rangle, \oplus\langle\varphi_2\rangle, \ldots \oplus \langle\varphi_n\rangle \Rightarrow \ominus\langle\psi\rangle$

The Substructural Inferentialist Framework
Going Simply Substructural

The Basic Proposal: Accommodate our data *directly* by having a bilateral inferentialist framework that is *simply substructural*. Specifically, just don't globally impose the following principles:

$$\frac{\Gamma \Rightarrow A}{\Gamma, B \Rightarrow A}\ \text{Monotonicity (MO)} \qquad \frac{\Gamma \Rightarrow A \quad \Gamma, A \Rightarrow B}{\Gamma \Rightarrow B}\ \text{Cumulative Transitivity (CT)}$$

so we can simply take as basic:

- **Non-Monotonic Consequences:**
  - $\oplus\langle\textbf{grass}\rangle \Rightarrow \oplus\langle\textbf{green}\rangle$
  - $\oplus\langle\textbf{grass}\rangle, \oplus\langle\textbf{scorched}\rangle \not\Rightarrow \oplus\langle\textbf{green}\rangle$
- **Non-Cumulatively Transitive Consequences:**
  - $\oplus\langle\textbf{bird}\rangle \Rightarrow \oplus\langle\textbf{flies}\rangle$
  - $\oplus\langle\textbf{bird}\rangle, \oplus\langle\textbf{flies}\rangle \Rightarrow \ominus\langle\textbf{penguin}\rangle$
  - $\oplus\langle\textbf{bird}\rangle \not\Rightarrow \ominus\langle\textbf{penguin}\rangle$

The Substructural Inferentialist Framework

The Way Forward

- Just as in the truth-conditional framework, we'll start with the meanings of atomic sentences, *but*
  - Rather than being understood in terms of sets of worlds assigned to atomics, these meanings are going to be understood in terms of the *base consequence relation* in which atomics figure.
- Just as in the truth-conditional framework, the meanings of an infinite set of logically complex sentences will be recursively determined from the meanings of atomics, *but*
  - Rather than being determined by the repeated application of set-theoretic operations, these meanings are going determined by the repeated application of *proof rules*.
- Can we proceed in this fashion, ending up with all the good results and none of the bad ones?
  - Yes! And I'll show how, laying out new kind of proof system that does the job.

NM-B

Some Background

- Existing versions of bilateral logic (e.g. Smiley 1996, Rumfit 2000, Francez 2015) are all natural deduction systems that impose the structural rules of MO and CT.
- Substructural logic has generally been done with the use of *sequent calculi*, which require explicit appeal to structural rules:
  - Rather than having rules for deriving *sentences*, we have rules for deriving *sequents*, which have the form $\Gamma \Rightarrow A$ (or $\Gamma \Rightarrow \Delta$).
  - Rather than having *introduction and elimination* rules, we have *only introduction* rules.
- The space of *bilateral* sequent calculi, where sequents relate positively or negatively *signed* sentences is completely unexplored.
- The bilateral sequent calculus I'll introduce, which I'll call "NM-B" (for "Non-Monotonic Bilateral") is based on the multiple conclusion sequent calculus NM-MS ("Non-Monotonic Multi-Succident") proposed by Kaplan (2018).

NM-B
Axioms

- Rather than having only *logical* axioms we'll also have *material* axioms—namely, any of the consequences belonging to our base consequence relation:

$$\frac{}{\Gamma \Rightarrow A}\ \text{Material Base (MB)}$$

if $\Gamma \Rightarrow A$ is a base consequence.

- We'll also have an axiom that says, trivially, that if you occupy some set of normative positions, then you occupy any normative position in that set:

$$\frac{}{\Gamma, A \Rightarrow A}\ \text{Containment (CO)}$$

- In both of these schemas, we require that Γ and $\{A\}$ contain only (positively or negatively signed) atomic sentences.

NM-B
Structural Rules

- We'll take what goes on the left side of a sequent of the form $\Gamma \Rightarrow A$ to be a *set* of normative positions.
- So, it doesn't matter how many times a normative position appears on the left of a sequent—the sequent expresses the same scorekeeping principle:

$$\frac{\Gamma, A, A \Rightarrow B}{\Gamma, A \Rightarrow B} \text{ Contraction (CNT)} \qquad \frac{\Gamma, A \Rightarrow B}{\Gamma, A, A \Rightarrow B} \text{ Expansion (EXP)}$$

- It also doesn't matter the order in which normative positions on the left of a sequent:

$$\frac{\Gamma, A, B, \Delta \Rightarrow C}{\Gamma, B, A, \Delta \Rightarrow C} \text{ Permutation (P)}$$

NM-B
Structural Rules

- We'll impose one substantive structural rule, a generalized contraposition principle that Smiley (1996) dubs "Reversal:"

$$\frac{\Gamma, A \Rightarrow B}{\Gamma, B^* \Rightarrow A^*} \text{ Reversal (RV)}$$

  Where staring a signed formula yields the oppositely signed formula

- **Four Kinds of Cases:**

$$1.)\ \frac{\Gamma, \oplus\langle\varphi\rangle \Rightarrow \ominus\langle\psi\rangle}{\Gamma, \oplus\langle\psi\rangle \Rightarrow \ominus\langle\varphi\rangle} \qquad 2.)\ \frac{\Gamma, \ominus\langle\varphi\rangle \Rightarrow \oplus\langle\psi\rangle}{\Gamma, \ominus\langle\psi\rangle \Rightarrow \oplus\langle\varphi\rangle}$$

$$3.)\ \frac{\Gamma, \oplus\langle\varphi\rangle \Rightarrow \oplus\langle\psi\rangle}{\Gamma, \ominus\langle\psi\rangle \Rightarrow \ominus\langle\varphi\rangle} \qquad 4.)\ \frac{\Gamma, \ominus\langle\psi\rangle \Rightarrow \ominus\langle\varphi\rangle}{\Gamma, \oplus\langle\varphi\rangle \Rightarrow \oplus\langle\psi\rangle}$$

NM-B

Negation Rules:

- If occupying a set of normative positions Γ *precludes one from being entitled* to some sentence φ, then Γ *commits one* to its negation, $\neg\varphi$:

$$\frac{\Gamma \Rightarrow \ominus\langle\varphi\rangle}{\Gamma \Rightarrow \oplus\langle\neg\varphi\rangle}\ \oplus_{\neg}$$

- Captures Brandom's (1994, 2008) definition of the negation of a sentence φ as its "minimal incompatible," the sentence implied by every set of sentences incompatible with φ.
- Alternately, if Γ commits one to φ, Γ precludes one from being entitled to $\neg\varphi$:

$$\frac{\Gamma \Rightarrow \oplus\langle\varphi\rangle}{\Gamma \Rightarrow \ominus\langle\neg\varphi\rangle}\ \ominus_{\neg}$$

- These are just the introduction rules of Rumfitt's (2000) natural deduction system.

NM-B

Positive Conjunction and Negative Disjunction Rules:

- If a set of normative positions Γ commits one to φ, and Γ also commits one to ψ, then Γ commits one to $\varphi \wedge \psi$:

$$\frac{\Gamma \Rightarrow \oplus\langle\varphi\rangle \quad \Gamma \Rightarrow \oplus\langle\psi\rangle}{\Gamma \Rightarrow \oplus\langle\varphi \wedge \psi\rangle} \oplus_{\wedge}$$

- Dually, if a set of normative positions Γ precludes one from being entitled to ψ, and Γ also precludes one from being entitled to ψ, then Γ precludes one from being entitled to $\varphi \vee \psi$:

$$\frac{\Gamma \Rightarrow \ominus\langle\varphi\rangle \quad \Gamma \Rightarrow \ominus\langle\psi\rangle}{\Gamma \Rightarrow \ominus\langle\varphi \vee \psi\rangle} \ominus_{\vee}$$

- Once again, these are just the introduction rules of Rumfitt's natural deduction system.

NM-B

Negative Conjunction and Positive Disjunction Rules

- If, relative to a set of normative positions Γ, φ and ψ are incompatible in the sense that commitment to one precludes entitlement to other, then Γ precludes one from being entitled to $\varphi \wedge \psi$

$$\frac{\Gamma, \oplus\langle\varphi\rangle \Rightarrow \ominus\langle\psi\rangle}{\Gamma \Rightarrow \ominus\langle\varphi \wedge \psi\rangle}\ \ominus_{\wedge}$$

- Dually, if, relative to Γ, φ and ψ are such that being precluded from being entitled to one commits one to the other, then Γ commits one to $\varphi \vee \psi$.

$$\frac{\Gamma, \ominus\langle\varphi\rangle \Rightarrow \oplus\langle\psi\rangle}{\Gamma \Rightarrow \oplus\langle\varphi \vee \psi\rangle}\ \oplus_{\vee}$$

- These are just crucially distinct from the introduction rules proposed by Rumfitt.

NM-B

Negative Conjunction and Positive Disjunction Rules

- The standard Negative Conjunction and Positive Disjunction rules (Rumfitt, 2000):

$$\frac{\Gamma \Rightarrow \ominus\langle\varphi\rangle}{\Gamma \Rightarrow \ominus\langle\varphi \wedge \psi\rangle}\ \ominus_{\wedge}\text{-I} \qquad\qquad \frac{\Gamma \Rightarrow \oplus\langle\varphi\rangle}{\Gamma \Rightarrow \oplus\langle\varphi \vee \psi\rangle}\ \oplus_{\vee}\text{-I}$$

- **Problem for** $\ominus_{\wedge}$-I:
  - ✓ Commitment to "Sadie lays eggs" precludes entitlement to "Sadie's a mammal."
  - So, # Commitment to "Sadie lays eggs" precludes entitlement to "Sadie's a mammal, and she's a platyplus," (by $\ominus_{\wedge}$-I).
- **Not a problem for us:** Because commitment to "Sadie lays eggs" along with commitment to "Sadie's a platypus" doesn't preclude entitlement to "Sadie's a mammal."
- **But not ad hoc:** Our negative conjunction rule directly encodes incompatibility, and positive disjunction rule the dual notion, a sort of disjunctive syllogistic relation.

The Same Good Results

- "The ball is red" implies "The ball is not green":

$$\dfrac{\oplus\langle\textbf{red}\rangle \Rightarrow \ominus\langle\textbf{green}\rangle}{\oplus\langle\textbf{red}\rangle \Rightarrow \oplus\langle\neg\textbf{green}\rangle}\ \oplus_{\neg}$$

- "The ball is not colored" is incompatible with "The ball is red or blue"

$$\dfrac{\dfrac{\dfrac{\oplus\langle\textbf{red}\rangle \Rightarrow \oplus\langle\textbf{colored}\rangle}{\oplus\langle\textbf{red}\rangle \Rightarrow \ominus\langle\neg\textbf{colored}\rangle}\ \ominus_{\neg}}{\oplus\langle\neg\textbf{colored}\rangle \Rightarrow \ominus\langle\textbf{red}\rangle}\ \mathrm{RV} \qquad \dfrac{\dfrac{\oplus\langle\textbf{blue}\rangle \Rightarrow \oplus\langle\textbf{colored}\rangle}{\oplus\langle\textbf{blue}\rangle \Rightarrow \ominus\langle\neg\textbf{colored}\rangle}\ \ominus_{\neg}}{\oplus\langle\neg\textbf{colored}\rangle \Rightarrow \ominus\langle\textbf{blue}\rangle}\ \mathrm{RV}}{\oplus\langle\neg\textbf{colored}\rangle \Rightarrow \ominus\langle\textbf{red} \vee \textbf{blue}\rangle}\ \ominus_{\vee}$$

The Same Good Results

- " The ball is a primary color, and the ball is not red or blue" implies "The ball is yellow."

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\oplus\langle\textbf{primary color}\rangle, \ominus\langle\textbf{yellow}\rangle, \ominus\langle\textbf{red}\rangle \Rightarrow \oplus\langle\textbf{blue}\rangle}{\ominus\langle\textbf{yellow}\rangle, \oplus\langle\textbf{primary color}\rangle, \ominus\langle\textbf{red}\rangle \Rightarrow \oplus\langle\textbf{blue}\rangle}\,\mathrm{P}}{\ominus\langle\textbf{yellow}\rangle, \oplus\langle\textbf{primary color}\rangle \Rightarrow \oplus\langle\textbf{red} \vee \textbf{blue}\rangle}\,\oplus_{\vee}}{\ominus\langle\textbf{yellow}\rangle, \oplus\langle\textbf{primary color}\rangle \Rightarrow \ominus\langle\neg(\textbf{red} \vee \textbf{blue})\rangle}\,\ominus_{\neg}}{\ominus\langle\textbf{yellow}\rangle \Rightarrow \ominus\langle\textbf{primary color} \wedge \neg(\textbf{red} \vee \textbf{blue})\rangle}\,\ominus_{\wedge}}{\oplus\langle\textbf{primary color} \wedge \neg(\textbf{red} \vee \textbf{blue})\rangle \Rightarrow \oplus\langle\textbf{yellow}\rangle}\,\mathrm{RV}
$$

- Any classical implication or incompatibility will have a proof whose leaves are all CO instances.
- Standard structural rules such as transitivity can be assumed to hold for *persistent* sequents which can be made explicit as follows:
  - If, $\Gamma \Rightarrow A$ and, for all Δ, $\Delta, \Gamma \Rightarrow A$, then $\Gamma \Rrightarrow A$

None of the Bad Results

- The logical system conservatively extends a non-Monotonic and non-Cumulatively Transitive material base consequence relation, enabling us to maintain
- **Non-Monotonic Consequences:**
  - $\oplus\langle\textbf{grass}\rangle \Rightarrow \oplus\langle\textbf{green}\rangle$
  - $\oplus\langle\textbf{grass}\rangle, \oplus\langle\textbf{scorched}\rangle \not\Rightarrow \oplus\langle\textbf{green}\rangle$
- **Non-Cumulatively Transitive Consequences:**
  - $\oplus\langle\textbf{bird}\rangle \Rightarrow \oplus\langle\textbf{flies}\rangle$
  - $\oplus\langle\textbf{bird}\rangle, \oplus\langle\textbf{flies}\rangle \Rightarrow \ominus\langle\textbf{penguin}\rangle$
  - $\oplus\langle\textbf{bird}\rangle \not\Rightarrow \ominus\langle\textbf{penguin}\rangle$

Discursive Role Semantics
The Basics

- **The Key Idea:** To know the meaning of a sentence φ is how an utterence of φ can function to change the normative positions that a speaker occupies.
  - We'll think of speakers as having "scorecards" that say which normative positions they take the various other speakers to occupy.
  - These scorecards get *updated* by way of speakers' "scorekeeping principles" of committive and preclusive consequence.
  - Semantic values are assigned the style of dynamic semantics (Veltmann 1996): the semantic value of φ, from the perspective of some player m, will be a function that maps each scorcard m might have to the scorecard that'd result upon any player's uttering φ.

Some Points of Comparison

- **Objection:** One might think that possible worlds semantics *explains* relations of implications and incompatibility, and this is prima facie preferable to taking relations of implication and incompatibility as basic, as I have proposed we do here.
- **Reply:** However, a possible worlds semantics *also* must take relations of implication and incompatibility as basic, and it does so in a way that is less philosophically well-grounded and formally flexible than the approach suggested here.

Some Points of Comparison

- **Standard Definition of Worlds:** A possible world is a complete first order model, consisting in a set of objects, D, and an interpretation function, V, that maps individual constants to objects and n-place predicates to sets of n-tuples of objects.
- **The Problem:** These models need to be *restricted* in order for the semantics to work, so as to ensure, for instance, that, for any valuation function V, $V(\textbf{green}) \cap V(\textbf{red}) = \varnothing$
- **The Standard Solution:** Meaning postulates (Carnap 1956, Partee 2005):
  - $\forall x(\textbf{red}(x) \rightarrow \neg\textbf{green}(x))$
  - $\forall x \forall y((\textbf{red}(x) \wedge \textbf{pink}(y)) \rightarrow \textbf{darker than}(x, y))$
- The thought is that, it is through meaning postulates like this one, which determine the space of genuinely possible worlds in the compositional semantics, our compositional semantics encodes speakers' lexical knowledge.

Some Points of Comparison

- **But** . . .
  - Meaning postulates are implicitly modalized, but their modal content cannot, on pain of circularity, be understand in terms of possible worlds.
  - Only a fraction of the genuine lexical knowledge can be encoded in this way. For instance, the implication from "grass" to "green" gets excluded.
- **Resolved on Our Approach:**
  - Modal language can be understood as a "transposed language of norms," (Sellars, 1953).
  - Discursive role semantics is *intrinsically lexical*, and all lexical knowledge, including substructural knowledge is directly encoded.

Conclusion

- I obviously don't take myself to have shown the inferentialist approach is the only way to go here.
- But it is one way, quite an elegant one, and, right now, it's the only one on the table.
- So, at the very least, I take it that the inferentialist approach I've put forward here ought to be taken seriously.

Thank You!

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