

Summary of the Computational-Inferential Reading of Fregean *Sinn*

(This outline sketches Bob's answers to the questions raised about the distinctions rehearsed in the Handout for Week 6, "Frege's Mature Metavocabulary.")

1. The general idea is to understand:
 - i. *Bedeutung* as a sortal kind-term introduced by abstraction using the equivalence relation that assimilates expressions accordingly as they are intersubstitutable *salva veritate*.
 - ii. *Sinn* as a sortal kind-term introduced by abstraction using the equivalence relation that assimilates expressions accordingly as they are intersubstitutable *salva consequentia*. The account of *begrifflich Inhalt* from the *Begriffsschrift* is to be modified to include the *computational* story about inferential roles outlined below.
2. Understanding *mode of presentation* [*Art des Gegebenseins*]:
Passage from opening of *USB*:
"It is natural, now, to think of there being connected with a sign (name, combination of words, letter), besides that to which the sign refers, which may be called the reference [*Bedeutung*] of the sign, also what I should like to call **the sense of the sign, wherein the mode of presentation is contained.**" *Bedeutungen* are what modes of presentation present or specify.

Claim 1: Modes of presentation specify *Bedeutungen* as the *values* of *functions* for particular *arguments*. In this precise sense, they are *computations* of those values—ways of computing them. The values (*Bedeutungen*) are exhibited ('presented') as the results of applying some function to some arguments.

16 is the result of applying the function of raising 2 to an exponent, $\lambda x(2^x)$, to the argument 4. That function, applied to that argument, is a way of computing the value 16. It is a 'mode of presentation' of that object, which is the *Bedeutung* of the expression '2⁴'.

The expression 'morning star', understood as the definite description 'last heavenly body visible at sunrise' specifies Venus (its *Bedeutung*) as the result of applying the function 'last heavenly body visible at...' to the argument 'sunrise'. That is a different mode of presentation of Venus than 'the evening star', understood as the definite description 'first heavenly body visible at sunset' specifies Venus as the result of applying the different function 'first heavenly body visible at...' to the different argument 'sunset'.

Shorthand: modes of presentation are functions appplied to arguments, or f-a-t-a s.

That is just what computations are, with the values resulting from applying the function to the arguments being the result of that computation. This is one of the basic ideas behind the Curry-Howard correspondence.

3. Understanding *sense* [*Sinn*]:

Senses are structured constellations of functions-applied-to-arguments.

The passage from *USB* cited above says that senses *contain* [*enthalten*] modes of presentation. (Note that the values of functions do *not* ‘contain’ the functions or arguments used to compute them. Neither Sweden nor the function *capital of* are contained in Stockholm.)

Claim 2: The sense of an expression consists of *all* of the substitutional analyses or decompositions of it into functions-applied-to-arguments. All of these must be ways of computing the *same* value or *Bedeutung*.

Frege has emphasized since *BgS* that expressions can be functionally analyzed in multiple ways. For instance, the expression ‘2⁴’ can be analyzed as:

- i. The result of applying the function $\lambda x(2^x)$ to the argument 4, and
- ii. The result of applying the function $\lambda y(y^4)$ to the argument 2, and
- iii. The result of applying the function $\lambda x,y(y^x)$ to the arguments $\langle 4,2 \rangle$.

All these are modes of presentation of the same *Bedeutung*, ways of computing the same value, namely 16. All three of those modes of presentation of 16 are *contained in* the *sense* of the expression ‘2⁴’.

Frege says: “This will be surprising only to **somebody who fails to see that a thought can be split up in many ways, so that now one thing, now another, appears as subject or predicate**. The thought itself does not yet determine what is to be regarded as the subject. If we say ‘the subject of this judgment,’ we do not designate anything definite unless at the same time we indicate a definite kind of analysis.” [Concept and Object, 49].

One of the central themes of Danielle Macbeth’s *Frege’s Logic* is that the *Begriffsschrift* is a notation that perspicuously displays *all* the functional analyses of each of its expressions, without privileging any one of them (as our standard one-dimensional notation obliges us to do for complex logical sentences).

The f-a-t-a s (i), (ii), and (iii) are not just a set. They come with a structure. The substitutional equivalence class of expressions corresponding to the function $\lambda x(2^x)$ (which includes ‘2³’, ‘2⁴’, ‘2⁵’ and so on) and the substitutional equivalence class of expressions corresponding to the function $\lambda y(y^4)$ (which includes ‘1⁴’, ‘2⁴’, ‘3⁴’ and so on) are subsets of the substitutional equivalence class of expressions corresponding to the function $\lambda x,y(y^x)$, which is their union.

4. Senses and inferential roles:

Claim 3: **Senses**, understood as constellations of modes of presentation of the *Bedeutungen* that are the values computed by the functions-applied-to-arguments contained in the senses, **determine the inferential roles** of the expressions whose senses they are—paradigmatically, the thoughts expressed by declarative sentences.

The function $\lambda x(\text{Walks}(x))$, which has the True as its value if the *Bedeutung* of the expression substituted for x walks, stands in the inferential relation to the function $\lambda x(\text{Moves}(x))$ that is made explicit by a quantified conditional $\forall x[\text{Walks}(x) \rightarrow \text{Moves}(x)]$. Corresponding to each functional decomposition of an expression is a pattern of relations of implication that applications of that function exhibit with respect to instances of other functions.

5. Cognitive values:

Frege says at the outset of *USB* “ $a=a$ and $a=b$ are obviously statements of **differing cognitive value**; $a=a$ holds *a priori* and, according to Kant, is to be labelled analytic, while statements of the form $a=b$ often contain very **valuable extensions of our knowledge** and cannot always be established *a priori*. The discovery that the rising sun is not new every morning, but always the same, was one of the **most fertile** (*folgenreich*) astronomical discoveries.” [USB 56]

Claim 4: The inferential roles determined by all the modes of presentation (f-a-t-a s) comprised by the sense of an expression are its *cognitive value* (*Erkenntniswert*).

Frege started off *BgS* by defining two sentences as having the same *conceptual content* just in case, when combined with the same set of collateral premises or auxiliary hypotheses, they have the same inferential consequences. We can understand his observation at the beginning of *USB* that sentences of the form ‘ $a=b$ ’ have a different cognitive value from sentences of the form ‘ $a=a$ ’ as registering that they play different roles as premises of the inferences determined by their senses. For instance, from ‘ $a=b$ ’ and ‘ Pa ’, the conclusion ‘ Pb ’ follows—as well as ‘ Pa ’. But from ‘ $a=a$ ’ and ‘ Pa ’, only ‘ Pa ’ follows. (Identities, we learned in *GL*, are intersubstitution licenses.) This is the sense in which an identity of the form ‘ $a=b$ ’ can be ‘fertile’ in the sense of ‘rich in consequences’ (*folgenreich*).

The identity ‘ $2^4=4^2$ ’ equates the *Bedeutung* computed by all the f-a-t-a s contained in the sense of ‘ 2^4 ’ with the *Bedeutung* computed by all the f-a-t-a s contained in the sense of ‘ 4^2 ’. The key to understanding the role played by the metaconcept of cognitive value in motivating the distinction between sense and *Bedeutung* at the beginning of *USB* is understanding the difference between the relations between any two f-a-t-a s contained in the sense of ‘ 2^4 ’ and the relations between any one of those f-a-t-a s and any f-a-t-a contained in the sense of ‘ 4^2 ’—even though all of them are computations yielding the same value, namely 16.