

Early Arguments from *Grundlagen*

Harold Bloom: “Every strong reading is a rewriting.”

You might think Frege’s topic is *numbers*.

In fact, he uses that concern to address a topic that is *much* more general:

Frege

- a) develops substantially **new metaconcepts of object and concept**.

That is, he

- b) develops a new vocabulary (**inferential** and **substitutional**: the two biggest moves he makes) expressing metaconcepts of object and concept.

Intro:

Look back at part of *BgS* that we did not talk about last time: the treatment of singular terms.

Read and discuss §8 [Add this to Handout for Week 3]:

§ 8. **Identity of content** differs from conditionality and negation in that it applies to names and not to contents. Whereas in other contexts signs are merely representatives of their content, so that every combination into which they enter expresses only a relation between their respective contents, they suddenly display their own selves when they are combined by means of the sign for identity of content; for it expresses the circumstance that two names have the same content. Hence the introduction of a sign for identity of content necessarily produces a bifurcation in the meaning of all signs: they stand at times for their content, at times for themselves. At first we have the impression that what we are dealing with pertains merely to the *expression* and *not to the thought*, that we do not need different signs at all for the same content and hence no sign whatsoever for identity of content. To show that this is an empty illusion I take the following example from geometry.

Assume that on the circumference of a circle there is a fixed point *A* about which a ray revolves. When this ray passes through the center of the circle, we call the other point at which it intersects the circle the point *B* associated with this position of the ray. The point of intersection, other than *A*, of the ray and the circumference will then be called the point *B* associated with the position of the ray at any time; this point is such that continuous variations in its position must always correspond to continuous variations in the position of the ray. Hence the name *B* denotes something indeterminate so long as the corresponding position of the ray has not been specified. We can now ask: what point is associated with the position of the ray when it is perpendicular to the diameter? The answer will be: the point *A*. In this case, therefore, the name *B* has the same content as has the name *A* ; and yet we could not have used only one name from the beginning, since the justification for that is given only by the answer. One point is determined in two ways:

(1) immediately through intuition and (2) as a point *B* associated with the ray perpendicular to the diameter.

To each of these ways of determining the point there corresponds a particular **name**. Hence the need for a sign for identity of content rests upon the following consideration: **the same content can be completely determined in different way**; but that in a particular case *two ways of determining it* really yield the *same result* is **the content of a judgment**. Before this judgment can be made, two distinct names, corresponding to the two ways of determining the content, must be assigned to what these ways determine. The judgment, however, requires for its expression **a sign for identity of content, a sign that connects these two names**. From this it follows that the existence of different names for the same content is not always merely an irrelevant question of form ; rather, that there are such names is the very heart of the matter if each is associated with a different way of determining the content. In that case the judgment that has the identity of content as its object is synthetic, in the Kantian sense. A more extrinsic reason for the introduction of a sign for identity of content is that it is at times expedient to introduce an abbreviation for a lengthy expression.

Then we must express the identity of content that obtains between the abbreviation and the original form.

Now let

$$\vdash (A = B)$$

mean that ***the sign A and the sign B have the same conceptual content, so that we can everywhere put B for A and conversely.*** [§8]

Here we see the idea that the *content* of the singular term is the *object* that it refers to, the *bearer* of the *name*. (Frege's word for singular terms is *Eigenname*, literally, "proper name."). He understands identity of bearers *as* licensing intersubstitution *salva consequentia*—*not* *salva veritate*.

Noting invariance under substitution is assimilating *arguments* insofar as they share a *value* for some function. If the arguments are 'names', that is, singular terms, and the values are the conceptual contents expressed by declarative sentences, then the functions are *concept-expressions*, that is, predicates.

So here we have a theory of *objects* and *concepts*, in the form of an account of the conceptual contents of *names* and of functions from the conceptual contents of names to conceptual contents of judgments.

Might briefly rehearse some other important kindsof singular terms:

definite descriptions, and "token reflexives" (Reichenbach's useful term, downstream from Peirce's distinction between types and tokens) indexicals, demonstratives—*not* to be assimilated syntactically to indexicals—and anaphoric pronouns.

Frege's thought is that singular terms with the same referent or bearer can be substituted for one another (intersubstituted) *salva consequentia*.

They can, in *some* contexts: extensional. And we *can* make this a criterion for the terms having *primary occurrence*. But in epistemic and modal contexts, this is not so. In *USB* Frege emphasizes the different *epistemic* status of ‘a=a’ and ‘a=b’. But he could just as well have appealed to *alethic modal* contexts: it is necessarily true that a=a, but not so for various instances of a=b, for instance, when one is a definite description. And I have already emphasized (with the passage from the Preface on extensions of Begriffsschrift to physics) that he means to capture lawlike and lawful relations among concepts.

This was Frege’s first try at extending the inferential role notion of conceptual content to subsentential expressions. He could *just* have appealed to intersubstitutability *salva consequentia*. But he *adds* the identification of the conceptual content of a term with the point picked out by the “ways of determining the content.” That introduces the name/bearer model. His **next try at integrating the semantic two models** is the first half of *Grundlagen*, the part we are about to discuss.

1. Numerals are singular terms (Quine: “purporting to refer to just one thing”).

When we ask someone what the number one is, or what the symbol one means [bedeute], we get as a rule the answer "Why, a **thing**". [Introduction, p. i]

2. They answer the question “How many?”

Compare: Anscombe on agency defined by answers to one kind of “Why?” question.

Actions as what we can appropriately ask “Why?”.

The first key question will turn out to be:

Q: **What is it we can ask “How many?” about?**

A: Anything. Objects, particulars, things, entities....

Here Frege invokes, and will later rely on, the deep connection between *numerals/numbers* and the activity of *counting*.

Some writers define Number as a set or multitude or plurality. All these views suffer from the drawback that the concept will not then cover the numbers 0 and 1. [28]

It is no good objecting that 0 and 1 are not numbers in the same sense as 2 and 3. What answers the question "How many?" is number, and if we ask, for example, "How many moons has this planet?", we are quite as much prepared for the answer 0 or 1 as for 2 or 3, and that without having to understand the question differently. [44]

So 0 and 1 are licit numerals, and any account must explain them.

Worse, if we use the Leibniz strategy of defining number, in general, by appeal to 1 (or, we can now see, better 0) and to *increase by one*, getting these is uniquely important.

3. Looking for the **unity** of units, that is, of countables.

‘Haecceity’ is another term. ‘Thing’, ‘something’, ‘being’, ‘entity’, ‘object’, particular....

(See discussion of pseudosortals below)

What *property* must something (?) have in order to be countable, to serve as a unit for counting.

Does the number word [Zahlwort] 'one' stand for a **property** of objects?... It must strike us immediately as remarkable that every single thing should possess this property. It would be incomprehensible why we should still ascribe it expressly to a thing at all. It is only in virtue of the possibility of something not being wise that it makes sense to say "Solon is wise." The content of a concept diminishes as its extension [Umfang] increases; if its extension becomes all-embracing, its content must vanish altogether... In isolation it seems that 'one' cannot be a predicate [Praedicat]. This is even clearer if we take the plural. **Whereas we can combine "Solon was wise" and "Thales was wise" into "Solon and Thales were wise", we cannot say "Solon and Thales were one". But it is hard to see why this should be impossible, if 'one' were a property both of Solon and of Thales in the same way that 'wise' is.** [s29;p40-41]

This is looking for a **property of unity** that distinguishes everything from everything else.

Cf. “Everything is what it is, and not some other thing.”

Question is how to make this *true*.

Every attempt to define 'one' as a property having thus failed, we must finally abandon the view that in designating a thing a unit we are adding to our description of it [eine nähere Bestimmung zu sehen [34]

Note (looking forward to discussion of sortals) that this issue is taken up again, in effect, by *analytic metaphysicians*, in the form of the search for (what we can now call) *ultimate* sortals. See “**Historical Context**” discussion below, of development from Aristotelian notions of substance (vs. “attributes”), through Frege, to losing category of sortals in classical analytic philosophy, relying on *Principia Mathematica*-style first-order logics, and Tarskian model theory (and the combination of those Quine articulated and theorized), to the recovery of the concept in analytic metaphysics—tied to a dubious philosophical agenda.

The “North Sea” example is relevant here.

4. The notion of ‘**unit**’ (or object, thing, particular, entity), Frege’s *Einheit*, is given two incompatible jobs.

Solution: as above: hidden relativity to an argument. See below.

The problem with ‘units’:

The symbols 1', 1", 1''' tell the tale of our embarrassment. **We must have identity--hence the 1;**

but we must have difference--hence the strokes; only unfortunately, the latter undo the work of the former... It follows, therefore, that on his view there would not only be distinct ones but also distinct twos and so on; for $1'''+1''''$ could not be substituted for [vertreten] $1''+1'''$. [36]

Problem of ‘units’: they must combine *identity* and *difference* in a way that can seem paradoxical.

(Cf. Hegelian and after ways of thinking about ontology in terms of identity and difference.)

5. How are we to curb the arbitrariness of our ways of regarding things [die Willkür der Auffassung], **which threatens to obliterate every distinction between one and many?** [45]

It is quite true that while I am not in a position, simply by thinking of it differently, to alter the color or hardness of a thing in the slightest, I am able to think of the Iliad either as one poem, or as 24 books, or as some large number of verses... If I give someone a stone with the words: Find the weight of this, I have given him precisely the object he is to investigate [dem ganzen Gegenstand seiner Untersuchung]. [BB: But not the units, e.g. pounds, kilograms, or grams.] But if I place a pile of playing cards in his hands with the words: Find the Number of these, this does not tell him whether I wish to know the number of cards, or of complete packs of cards, or even say of honour cards at skat. To have given him the pile in his hands is not yet to have given him completely the object he is to investigate; I must add some further word--cards, or packs, or honours... The number 1, on the other hand, or 100 or any other Number, cannot be said to belong to the pile of playing cards in its own right, but at most to belong to it in view of the way in which we have chosen to regard it...[s22;p28-29]

Objects	Linguistic Expressions	Activity
Decks	1: Numeral (singular term) + Sortal	Counting
Suits	4: Numeral (singular term) + Sortal	Counting
Cards	52: Numeral (singular term) + Sortal	Counting
Corners	208 Numeral (singular term) + Sortal	Counting
Numbers	Numerals (singular terms)	Counting

Sometimes, multiple apparently incompatible (after all, $1 \neq 4 \neq 52 \neq 208$) number statements seem to be objectively true.

Compare: The item weighs 1 kilogram, but also 2.2 pounds.

So obey the **Scholastic maxim**: “**When faced with a contradiction, make a distinction.**”

Mathematician used to functions says rather (encompassing the Scholastic maxim as a special case), there is a missing argument.

The value of the numbering function depends on a variable not yet acknowledged.

The value of function f is 1, and

The value of function f is 4,

The value of function f is 52, and

The value of function f is 208,

are not incompatible **if f has an argument**: decks, suits, cards, and corners.

Think of this thinking in terms of functions (that could have arguments like decks, cards, suits) as a *new* way of thinking (Frege's), adapted from the previous, still new mathematical study of functions as such.

Compare: The item weighs 1 kilogram, but also 2.2 pounds.

[Excursus on why '**parameter**' is not apt here. Parameters are coefficients of functions, not their arguments. They, too, can be varied, but that is a different, second-order issue. In $s = 1/2 gt^2$, g is a parameter and t is the variable=argument of the function. Begriffsschrift purposely allows discerning both the first-order and the second-order relations here.]

Objectivity as a Criterion of Adequacy:

For number is no whit more an object of psychology or a product of mental processes than, let us say, the North Sea is. **The objectivity [Objectivität] of the North Sea is not affected by the fact that it is a matter of our arbitrary choice which part of the water on the Earth's surface we mark off and elect to call the 'North Sea'...** The botanist means to assert something just as factual when he gives the Number of a flower's petals as when he gives their color. The one depends on our arbitrary choice [Willkür] just as little as the other. There does, therefore, exist a certain similarity between Number and color; it consists, however, not in our becoming acquainted with them both in external things through the senses, but in their both being objective. [s26;p34]

6. That missing argument, that (4) and (5) above give reasons to seek, had better not be anyone's *subjective ideas or conceptions*. For some number claims are objectively true or false, in the sense of being true or false independently of (= not as a function of) anything merely subjective.

Those arguments set the principle criteria of adequacy for an account of number.

How does Frege propose to satisfy them?

7. Units are concepts.

Concepts are the "ways of regarding things."

The concepts things *fall under* (not what concepts "contain") provide the *unity* of 'units'.

The different things that fall under the concept provide the differences, and are what can be *counted*.

Why not, in fact, adopt this very apt suggestion, and **call a concept the unit relative to the**

Number which belongs to it? [...Einheit zu nennen in Bezug auf die Anzahl welche ihm zukommt] [54]

Now that we have learned **that the content of a statement of number is an assertion about a concept...**[55]

Conclusion of this line of thought is that the subject of number claims, that to which numbers are attributed, is *concepts*.

What number-phrases (of which numerals are just parts) are predicated of, what is numbered, what numbers are assigned to, is what falls under concepts.

Paradigm: **the number of Jupiter's moons is 4.**

(These are the "Galilean moons." Today we would say the number of Jovian moons is 79.)

We realize that what "units" are, which seem to have contradictory properties, until we see how to reconcile them—a move of the kind Hegel offered a metavocabulary to articulate (epitomized in *Schelling's* thesis-antithesis-synthesis)—is just concepts.

Lots of concepts will do: pack, card, suit....

8. **Concepts can be objective.**

(Might call our "ideas" of them '*conceptions*.')

Number claims (the number of moons of Jupiter is 4) involve a sometimes-hidden relativity or dependence. It would be bad if it were to our subjective attitudes or thinkings.

Fortunately, we can understand it as relativity to *concepts*.

Concepts must accordingly be taken to be what numbers are in the first instance *about* or *assigned to*.

This understanding of the relativity of number claims to concepts is compatible with the *objectivity* of (at least some) number claims, in the sense of their *independence* of the subjective acts or attitudes of thinkers.

In this connection, we can see the importance of **Frege's remark that the North Sea is no less objective a thing because we can make different decisions about how to draw boundaries around it.**

For number is no whit more an object of psychology or a product of mental processes than, let us say, the North Sea is. **The objectivity [Objectivität] of the North Sea is not affected by the fact that it is a matter of our arbitrary choice which part of the water on the Earth's surface we mark off and elect to call the 'North Sea'...** The botanist means to assert something just as factual when he gives the Number of a flower's petals as when he gives their color. The one depends on our arbitrary choice [Willkür] just as little as the other. There does, therefore, exist a certain similarity between Number and color; it consists, however, not in our becoming acquainted with them both in external things through the senses, but in their both being objective. [s26;p34]

Here I want to say that the principal fact is that we can compound concepts so as to find, for any two ‘things’, an *infinite* number of respects of similarity between them, and an infinite number of respects of *dissimilarity* between them. (We might need to gerrymander a little, in the way we do with Cambridge properties and Cambridge changes.) There are two sorts of extreme attitudes one might have towards this fact:

a) Pragmatists, like Rorty, conclude that what *privileges some* of those respects of similarity/dissimilarity, makes them more important or significant, must be relation to *our* practices and form of life.

b) Ontologists, such as David Lewis, say that there must be some *intrinsic, objective* privileging of some of those respects of similarity/dissimilarity. Some must be more *natural* than others, must “cut nature at the joints” in a way that is intelligible in principle as independent of anything about our discursive activity.

Frege’s “North Sea” example is an important data point that both of the extreme views should attempt to accommodate.

For Frege, numbers are a *paradigm* of *objects*.

So he needs an account of what is required to refer to objects that works for numerals referring to numbers.

Part II (of my presentation): Sortals, Identity, and Substitution [vertreten].

You might think that the answer to the question:

What is it that we can ask “How many?” questions about?

is *sets*.

That is, “How many?” questions are always about how many *elements* some *set* has.

The trouble is that ‘elements’ are like ‘units’ or ‘objects’.

Sets are the same if all their elements or members are the same.

So the question is put off to the counting of elements.

But we need to know how to identify and individuate those elements in order to count them.

9. **Not all concepts permit enumeration, numbering**, or counting, in the sense of answering the question “How many?”.

Q: Which do?

A: **Sortal concepts**

a) But not *all* concepts are suitable to determine *countable units*.

Only concepts that “isolate what falls under them in a definite way.”

What are those concepts, and how can we pick them out (the class of concepts that do that)?

The ultimate answer is: *sortal* concepts. (That is not one of Frege’s words.)

Paradigms of these are Aristotelian, Scholastic, and Early Modern *substantial* concepts: concepts of *substances*. Green is an *attribute*. Frogs are substances, and can *have* attributes.

Substances can be counted, attributes only in a different sense: “How many frogs on the log?” is a sensible question. Even if they are all green, “How many greens on the log?” would be asking about *shades* or *kinds* of green, or *spots* of green, or something like that.

Q: What is the difference?

The proposition asserting that units are isolated and indivisible can, accordingly, be formulated as follows: **Only a concept which isolates what falls under it in a definite manner, and which does not permit any arbitrary division of it into parts, can be a unit** relative to a finite Number. [54]

All predicates have circumstances (sometimes marked as ‘criteria’ of application. and consequences of application. (Sometimes these are picked out as ‘criteria’ of application. I think this is misleadingly explicit: criteria can be *stated*, circumstances of application might be practically mastered without being statable by practitioners—or even by theorists. And I think it

leaves out the inferential consequences downstream.) Merely characterizing predicates, like ‘green’ have *only* such circs and cons of app.

But **sortal concepts**, as opposed to *merely characterizing* concepts (and for this discussion we are thinking of monadic predicates, not relational ones), have, in addition to criteria or circs and cons of application, also **criteria (see reservation above) of identity and individuation**. They “isolate what falls under them in a definite way” in that they (their contents) supply an answer to questions of the form “Is this *the same* K as that?” I think this terminology of “criteria of application” and “criteria of identity and individuation” is Quine’s, from *Word and Object*.

The notion of “isolating what falls under them in a definite way” is cashed out in terms of supplying answers to this question: determining what it is to “recognize a K as the same again.” This demand comes from thinking about what is needed for the practical activity of **counting**, which is what one must do to answer the question “How many?”. If the K one assigns to the numeral ‘1’ when counting is the same as the K one assigns to a later number, one will have *miscounted*. (Leaving one out, assigning *no* number to it, is a *different* kind of mistake.) To count Ks one must be able to *tell them apart*—so as to be able to assign *different* numerals. And one must be able to *recognize a K as the same again*—so as *not* to assign *different* numerals to it. So to set a *normative standard* for determining whether one has counted Ks *correctly*, the concept K must settle the ‘criteria’ [see reservation above] of *identity* (sameness) and *individuation* (difference).

That is what makes sortal concepts K suitable to serve as *units* for counting: the concept determines what all the ‘units’ have in *common*, and the *differences* between Ks determines what are different *instances* of that ‘unit’.

10. **Mass nouns:**

Also here we can consider the contrast class of **mass nouns, such as ‘water’ and ‘gold’**.

Here to count them, to get a concept of the right sort, we must *complete* the phrase, as we do with ‘shades’ or ‘regions’ of green. We need to add *classifiers*, such as ‘cups’ of water, ‘pounds’ of water, ‘liters’ of water’, and so on. *These* concepts are concepts of countables, even though ‘green’ and ‘water’ *by themselves* are not.

Mention (following Quine in *Word and Object*, which is good on this general topic) the example of **Japanese classifiers**: one uses a variant of “piece of...” as though things were mass nouns, but with different ones for things that are flat and thin (paper pages, leaves) or long round and thin (pencils, chopsticks).

This generalizes and is a more determinate and articulated version of Kant’s treatment of objects as regions of space (and time). (See “Historical Context” below.)

Mass nouns + classifiers *also* form *common nouns*, that can identify and individuate *countable* things. But also,

measuring : counting :: mass nouns : proper sortals.

Both are *terms*: **general terms**, as opposed to *singular terms*.

Note that **children learn sortal predicates**, that is, *common nouns*, before they learn *attributive predicates*. Red and square are *difficult*, relatively *late-coming* concepts for them—against everything British Empiricists claimed and expected. **Dog and cat come very early**. Sortal concepts, concepts of *substances*, come *much* more naturally to them than attributive-adjectival concepts.

Progression of argument:

A) So far: Not all predicates, i.e. things that can be said of objects = applied to singular terms. **Not red, heavy, far away...**

These, like genuine sortals “isolate things that fall under them in a definite way”

That is (first sharpening), they don’t just have **criteria of application, but also criteria of identity and individuation**. **Cats and mats. Frogs and logs.**

B). Second sharpening of “isolating...” by sortals: making things countable, by **settling what counts as recognizing a K as the same again**. This gets both aspects: **identity and individuation**, and explains why they go together. If you could only get the *nonidentities* right, that need not settle the identities, and vice versa. Requiring that *both* be settled is an important step towards seeing that what is needed-wanted is *sets* of identity claims, understood to be exhaustive. (This is an important datum for (E) below, on how we settle *which* sets of identities need to be settled.)

Sortal concepts: The proposition asserting that units are isolated and indivisible can, accordingly, be formulated as follows: **Only a concept which isolates what falls under it in a definite manner, and which does not permit any arbitrary division of it into parts, can be a unit** relative to a finite Number. [54]

Numbers (numerals) answer the question “How many?”. That characterization of them connects them with the activity of *counting*, on the side of *pragmatics*.

For to answer a “How many?” question, what one does is count.

Thinking of what **counting** is, and what is required for it, leads us to the twin demands that sortal concepts *identify* and *individuate* things falling under them, that is, settle when two specifications are of the *same* K, and when they are of *different* Ks. That is a presupposition of countability.

For one must both be able to tell when two K’s are the same (criteria of identity), and when two K’s are different (criteria of individuation).

The search for a notion of unity shared by everything countable as such is asking for an attributive property that is shared by all and only countables—that is, we now see, everything that can fall under *any* sortal.

11. **Historical Context:**

- i. For sortals: **Aristotle** starts with sortals, which correspond to substances.

(Mass nouns, like water and gold get a secondary place.)

- ii. **Early moderns**, think of **Spinoza**, also cared a lot about substances, but didn't come close to this kind of understanding of them.

- iii. **Kant:**

Moved from traditional *Aristotelian* ways of identifying and individuating (material, empirical) objects, in terms of *substances* (picked out by sortals), to *Newtonian* criteria of identity and individuation, in terms of space and time. This is to identify objects by *spatial* (eventually, spatio-temporal) *regions*.

That is in terms of *mass* nouns: space (and time), plus *classifiers*.

That is, this a kind of gunk-world picture.

This transformation is an under-appreciated theme of the **Transcendental Aesthetic**.

Ernst **Cassirer** makes a lot of this Kantian transformation in his (also under-appreciated today) early TwenCen *Substance and Function*—whose index example is the transformation from *anatomy to physiology*, from thinking about biological creatures in terms of *structures* to thinking about them in terms of *functions*.

I *think*, but cannot testify to it as a fact, that Frege's teacher **Kuno Fischer**—who read Kant as first and foremost a philosopher of science, specifically, of Newtonian physics, and also founded neo-Kantianism, with his slogan “Zurück nach Kant”—is the first to read Kant as moving to Newtonian criteria of identity and individuation. (Hume wanted to be “the Newton of the mind,” but he did *not* understand the *math* that made Newtonian physics work.)

His criticism of substance concepts gives aid and comfort to a view that sees all predicative concepts as having the form of *attributive* predicates.

His concept of intuition accordingly gives us particulars without sortals.

This gives aid and comfort to later analytic philosophy, using a *Principia Mathematica* style logic.

- iv. Frege. I think he probably should be understood as having a Wiggins view (see below): identity is absolute, but terms come with sortals. Most cross-sortal identities are false (cat/kitten is a paradigm of where they are not), as can be seen if we think about *modal* properties—cf. lumps of clay and statues.

- v. Bertrand **Russell:**

Early on, distinguished, on the side of *terms* (sortal and singular), metaphysically between *bucket of shot and bowl of jelly* pictures: atomistic and holistic or gunk-world, with individuation in the latter being by *classifiers*.

(Cf. **Japanese classifiers**, as per Quine in *Word and Object*.)

Russell saw himself as moving from a *property*-oriented picture to a *relational* picture (as did Peirce, who called his logic a “logic of relations.”)

That is because he thought of moving from *monadic* to *polyadic* predicates.

But he thought of those predicates as purely *attributive*.

As in contemporary first-order logic (as opposed to more traditional *term logics*—which Geach championed), sortal restrictions on quantifiers (or the underlying terms) are handled with *attributive* conditions:

$\forall x(\text{Dog}(x) \rightarrow \text{Mammal}(x))$ is on a par with $\forall x(\text{Red}(x) \rightarrow \text{Colored}(x))$.

Everything is supposed to be handled with *attributive* predicates and *identity* claims.

Frege's *notation* invites this reading, but does not demand it.

And the *GL* argument speaks against such a reading, in favor of a more Wiggins-like one.

- vi. Analytic philosophy, under the influence of the first-order predicate calculus with identity, did not distinguish sortal predicates from mass terms or merely attributive adjectives. This led to **Geach's sortal relativization of quantifiers**, and to (my, and MacFarlane, in his diss) critiques of Tarskian and Quinean model theory as outsourcing the individuation of domain elements to a more expressively powerful metalanguage, that they can in turn only understand model-theoretically, so putting the issue of (ultimate?) sortals off sequentially, in an unexplanatory, unexplicating regress..

John McDowell's undergraduate tutor at New College (the one who gave him the nickname "the man with the Rolls Royce mind", David Wiggins's *Sameness and Substance* is *locus classicus* for the history and contemporary significance of the substance concept.

- vii. **Analytic metaphysics:**

David Lewis is responding to this concern metaphysically, by postulating basic particles, and acknowledging mereological sums of them. (The trouble with this line is, that underproduces. Ordinary kinds of things, tables, cats... are not mereological sums of anything. The modal properties—which in fact individuate thing-kinds—are quite different.)

Gunk worlds are the result of taking mass nouns as the fundamental individuating devices. Atomistic worlds take sortals, plus the idea of an *ultimate* sortal, on the basis of which the rest can be derived.

Or it can be *properties* all the way down: only attributive adjectives, not noun-phrases of any sort, either sortal or mass noun.

In any of these cases, we can see big divides in analytic metaphysics corresponding to what one takes the ultimate *form* of individuation to be.

Metaphysics is the idea of an *ultimate* form of individuation.

This is the form of one of the big issues in contemporary analytic metaphysics

This is a big, quick, swooshy, *Geistesgeschichtlich* history, tracing one strand, substances-sortals, from Aristotle, through to Early moderns, Kant, Frege, and the degenerate form of the idea in

analytic philosophy, at least before its metaphysical turn—which is nontrivially a response to this flaw in early analytic phil.

Numbers are things. That settles one sort of judgment that must apply to them: judgments expressing the recognition of a thing as the same again. These recognition judgments are identity claims. They are to be understood ultimately as *intersubstitution licences*. That is what determines their essential *use*, on the side of *pragmatics*.

Numerals are singular terms (Quine: “purporting to refer to just one thing”).

When we ask someone what the number one is, or what the symbol one means [bedeute], we get as a rule the answer "Why, a **thing**". [Introduction, p. i]

These first two point in essentially the same direction: towards understanding sortal concepts as settling normative standards for identifying (recognizing as the same again) and individuating (recognizing as different) the objects that fall under those concepts.

Key transitional conceptualization:

Judgments expressing the recognition of an object as the same again.

These are what I am calling “recognition judgments.”

They are a *kind* of judgment (judgeable).

But we have already settled [festgestellt] that number words are to be understood as standing for self-subsistent objects. [dass unter den Zahlwoerten selbständige Gegenstände zu verstehen sind - that by number words, self-subsistent objects should be understood] And that is enough to give us a class of propositions which must have a sense, namely **those which express our recognition of a number as the same again**. [der Sätze, welche ein Wiedererkennen ausdrücken.] **If we are to use a symbol a to signify [bezeichnen] an object, we must have a criterion for deciding in all cases whether b is the same as a**, even if it is not always in our power to apply this criterion. [62]

In our present case, we have to define the sense [Sinn] of the proposition the number which belongs to the concept F is the same as that which belongs to the concept G that is to say, we must reproduce the content [Inhalt] of this proposition in other terms, avoiding the use of the expression the Number which belongs to the concept F In doing this, we shall be **giving a general criterion for the identity of numbers** [Kennzeichen für die Gleichheit von Zahlen]. **When we have thus acquired a means of [a] arriving at a determinate number and of [b] recognizing it again as the same**, we can assign it a number word as its proper name [zum Eigennamen geben]. [62]

(That it is *objects* that are being recognized as the same again, or not, is, at the next stage (C), the

warrant for thinking of it as being singular terms (“purporting to refer to individual objects”) that are related by recognition judgements—so, flanking the identity sign.)

The origin of all these requirements (isolating what falls under the concept in a definite way, identity *and* individuation, recognizing an object as the same again) is that to understand the sort of concepts that *numbers* attach to, we have to look to **what is required for counting**. This is a lesson Wittgenstein was slow to extract—but he got there eventually. It is a development of **Kantian pragmatism**: looking to what one must *do* (for Kant, synthesize a constellation of commitments with the rational unity distinctive of apperception) in order to understand *content*: judging to understand what is judged, better, judgeable.

What kind of judgments are recognition judgments?

C). Third sharpening: **Recognition judgments, as in (B) are identity claims.**

Identity claims.

Here the explanatory arc goes from “recognizing Ks as the same again” to identity claims as *recognition judgments*, to understanding that “in intersubstitution, all the laws of identity are contained.” That is, the result is understanding that in order to fix the content of the sortal concept K one must “fix the sense of recognition judgments”, which are identity claims, and then understanding those identity claims in terms of intersubstitution licenses.

This specifies the *inferential role*, and so the *begrifflich Inhalt* of identity claims.

To obtain the concept of Number, one must fix [feststellen] the sense of a numerical identity [Zahlengleichung]. [62]

Our aim is to construct the content of a judgement [den Inhalt eines Urtheils zu bilden] which can be taken as [auffassen läßt] an identity such that each side of it is a number. We are therefore proposing not to define identity specially for this case, but to use the concept of identity, taken as already known [des schon bekannten Begriffes der Gleichheit], as a means for arriving at that which is to be regarded as being identical. [63]

Singular terms (Eigennamme),

But we have already settled [festgestellt] that number words are to be understood as standing for self-subsistent objects. [dass unter den Zahlwoerten selbständige Gegenstände zu verstehen sind - that by number words, self-subsistent objects should be understood] And that is enough to give us a class of propositions which must have a sense, namely **those which express our recognition of a number as the same again.** [der Sätze, welche ein Wiedererkennen ausdrücken.] **If we are to use a symbol a to signify [bezeichnen] an object, we must have a criterion for deciding in all cases whether b is the same as a.** [62]

Identity claims (recognition judgments, i.e. judgments expressing the recognition of an object as the same again)

When we have thus acquired a means of [a] arriving at a determinate number and of [b] recognizing it again as the same, we can assign it a number word as its proper name [zum Eigennamen geben]. [62]

Individuating concepts *also* have *criteria of identity and individuation*.

(The *consequences* of identity and individuation are substitutional.)

To introduce a singular term, one must fix the sense of *identity* claims, those that express the *recognition* of an *object* as *the same again*.

Identity claims are *intersubstitution licenses*.

These amount to general **conditional** commitments:

Endorsing ‘a=b’ is a commitment to endorse Pa whenever one endorses Pb, and *vice versa*, for any sentence-frame P.

Substitution:

D). Fourth sharpening: **All the laws governing the use of identity statements as such are contained, to be understood in terms of the operation of, substitution (vertreten).**

Now Leibniz's definition is as follows: "Things are the same as each other, of which one can be substituted for the other without loss of truth." [Eadem sunt, quorum unum potest substitui alteri **salva veritate**.] This I propose to adopt as my own definition of identity. ...Now it is actually the case that **in universal substitutability [allgemeinen Ersetzbarkeit] all the laws of identity are contained**. [65]

E). Residual question (pointing to next time): *which* identities must have their truth values settled to introduce sortals and their terms?

F) Sortal relativity of identity?

Geach' issue of unrestricted vs. sortally restricted quantification arises here. (Subject of Anil Gupta's dissertation (with Belnap) and first book, *The Logic of Common Nouns*.)

People and passengers, persons and sur-persons.

The question is whether unrestricted quantification is so much as intelligible, which it is not if it depends on treating 'object' etc. as genuinely individuating sortals. It can *look* as though it does not, if one takes talk of model-theoretic *domains* at face value. (One should not.). If the only way out is to read it as quantifying over all possible sortals (surely it is not enough to restrict *that second-order* quantifier to *actual* sortals, in our vocabulary, or even what can be defined from them. (And do we allow abstraction? Then we quantify over all equivalence relations?). But then, by what means? Consider the kite of methods of generating new concepts from old in the kite of conceptual novelty. And then, I think, we are quantifying over all possible *vocabularies*. And I have principled objections to that idea. New vocabularies are in no way restricted to those

that can be algorithmically elaborated from old ones. This is a Wittgensteinian point. There can also be just plain new practices, or ones *practically* elaborated from old discursive practices.

12. Pseudosortals and prosortals:

Now that we know what sortals are, we can revisit the term ‘unit’.

We’ll see that it belongs in a box with the terms ‘object’, ‘thing’, ‘particular’, ‘individual’, and others.

These are Scholastic “transcendental universals” (properties).

We saw the trouble they get us into, with the “unity (being countable, is a special kind of **property**” view, at the beginning of *GL*.

The genus is really countable.

Are these genuine sortals?

No. They do not “isolate what falls under them in a definite way.”

They only stand in for genuine sortals.

a) What about terms such as ‘unit’, ‘object’, ‘particular’, ‘thing’...?

i. These terms are both indispensable and problematic.

For they are *not* themselves individuating sortal concepts.

One cannot *count* using them.

And yet they *stand for*, in some way, *all* the sortal terms that *can* be used to count.

ii. My conclusion is that they are best understood as *pseudo*-sortals, or, better, *prosortals*, we could also say “schematic” sortals—marking a position where a proper, that is, genuinely individuating, sortal goes. (The Scholastics called them ‘transcendental concepts.’)

iii. They are responsible for the sorts of confusion Frege patiently dissects in the first half of *GL*.

iv. Those *same* difficulties infect Tarskian model theory, including in its Quinean philosophical rendering. For the notion of a domain, is of a *set* of ‘objects’ or ‘elements.’ The set-theoretic concept of an element is one of the ‘transcendentals’. These domain elements, we are told, are “merely numerically distinct,” that is *all* we care about is that they *are* identified and individuated, so that they can be *counted*. But in specifying a domain for model-theoretic purposes, we do not appeal to any *actual* sortals. That is why model-theorists have to worry about “*Pythagorean*” models, where the domains consist just of numbers.

For we can *prove* that if there is a model that counterexamples a pattern of inference (and that is what models are for), then there is model whose domain consists entirely of numbers that does so. So how, then, can model theory pretend to offer an account of the *meanings* of *actual* concepts, that do *not* apply exclusively to, are not exclusively about, numbers?

13. **Counting**, as an activity—on to **Price's subject naturalism**.

Object naturalism about a vocabulary is naturalism about what one is talking *about*. It is the view that one can specify that, for some vocabulary, in a naturalistic vocabulary: perhaps that of fundamental physics, or the special sciences, or just ordinary empirical descriptive vocabulary.

Subject naturalism is naturalism about what one is *doing* in using the vocabulary.

The Fregean lesson Wittgenstein learns is that if there is nothing mysterious about *practices of counting*, what one is *doing* in *using numerals*—if *that* can be specified in a naturalistically acceptable vocabulary—then there should be no residual puzzles from the side of naturalism about numbers.

14. Kantian parallel 1:

Another strand is the arc from

- Kant's transcendental idealism, which understands ontological features of *nature* (empirical reality) in terms of the features of our cognitive activity that they reflect: paradigmatically, articulated by lawlike necessities because of features of the *inferential* articulation of the concepts we use to understand and represent empirical things. To
- Frege's addressing the question of whether *numbers* are *objects* by asking whether *numerals* are properly introduced *singular terms*, that is, whether the truth of identities involving them have been settled.

A Kant-Frege parallel:

Frege approaches the question of what numbers are by thinking about the presuppositions of the *act of counting*. This is a pragmatics to semantics order of explanation: attribute the content needed to support the activity.

This is the way I read Kant, as moving *from* the three task responsibilities that normatively govern the activity synthesizing a constellation of commitments with the rational unity distinctive of apperception *to* the sort of conceptual content judgeable contents must have. Frege focuses on re-identification, discriminating something as the same again, when "given or determined in different ways." Strawson does this for objects=particulars in *Individuals*, and Gareth Evans takes it further. By contrast to his Kant book, *Bounds of Sense*, this one is in effect Strawson's meditation on this part of the *Grundlagen*.

- Wittgenstein, as understood in terms of
- **Price's metaconcept of subject naturalism.**

The case of ***numbers and counting*** is a particularly good one to make this point, and for seeing it as a Wittgensteinian point.

If there is nothing mysterious about the *use of numerals in counting* (from which adding, multiplying, subtracting, and in general *arithmetic* uses are easy to elaborate), then any residual puzzlement we have about *numbers*, even within the confines of the strictest

naturalistic explicative-explanatory requirements, should be taken to show that there are problems with the framework in which we are understanding such things.

(This is a kind of *meta-methodological modus tollens* that is characteristic of Wittgenstein (Cf. the difficulty in reconciling the contingency of content of empirical concepts with their *rational* bindingness. For LW, that shows the bankruptcy of the traditional notion of rationality.)

- This line of thought adds to (or branches off from?) the one that starts with Aristotle on substances, passes through the Early Modern version, then drops this concept on the basis of the new logic in *Principia* form and model-theoretic form, and re-emerges in analytic metaphysics's principal concern with what *ultimate kinds* of things there are. This is the search for a sortal (or small set of sortals) such that all other (proper, genuine, 'real') kinds can be understood in terms of it.

15. Frege's further Kantian argument is that **what it means for numbers (or *anything*) to be properly semantically picked out as objects (=particulars) is for expressions referring to them (representing them) to be properly introduced so as to be *used as* singular terms.** That is to introduce them as having their indirect (from the *begrifflich Inhalt* of sentences in which they are arguments = can be *substituted for*) conceptual content determined by *identity* claims. Identity claims are, in turn, understood semantically (inferentially, their indirect inferential significance is) as intersubstitution licences.

That 'argument' is not made explicitly. It is not avowed as his strategy. We have to get it from what he *does*, not what he *says about* what he is doing. (Speaking for such a reading, consider the way in which the sense/ reference distinction is already implicitly in play in *GL*.)

[Along the way:

These sorts of arguments about what objects are is why you want to think philosophically.]