

## Understanding the Concept of Truth Values

Course materials available at: <https://spaceofreasons.netlify.app/courses/frege2025/>

Plan for unpacking the concept of *truth values*:

1. Three versions of a *basic discursive bipolarity*:  
Semantic: true/false, Pragmatic: accept/reject, and Rational-functional.
2. Two explanatory roles truth values play:  
Dummett's "free-standing" and "ingredient" conceptions of content.  
A notion of extensionality of the relation between them.
3. A formal model: synthetic multivalued logic.  
Distinguishing (ingredient) *multivalues* and (free-standing) *designatedness* values.
4. *Analytic*, substitutional use of the apparatus of multivalued logic.  
Semantic analogues of Lindenbaum algebras.
5. Relations and significance of some actual multivalued logics:  
K3 (Kripke), LP (Graham Priest's "Logic of Paradox"), ST (Strict-Tolerant), and FDE (First Degree Entailment).
6. Relating the *assertional* substitutional hierarchy (ingredient contents intersubstitutable *salva veritate*) to the *inferential* substitutional hierarchy (ingredient contents intersubstitutable *salva consequentia*) by shifting focus of pragmatic account of free-standing (designated) content.  
Another notion of extensionality, from the relations between these substitutional hierarchies.

Hlobil isomorphism:

- According to a bilateral, two-sorted normative **pragmatic** theory, for the premises  $\Gamma$  to *imply* the conclusion A is for anyone who *accepts* all of  $\Gamma$  to be precluded thereby from entitlement to *deny* A—and in that sense, to be *implicitly* committed to *accept* A.
- According to a modal-mereological truth-maker **semantic** theory, for premises  $\Gamma$  to *imply* the conclusion A is for every fusion of truth-makers of all the propositions in  $\Gamma$  with falsity-makers of A to be *impossible* states.

Frege:

We have seen that it is true of parts of sentences that they have *Bedeutungen*.

**What of a whole sentence, does this have a *Bedeutung* too?**

If we are concerned with truth, if we are aiming at knowledge, then we demand of each proper name occurring in a sentence that it should have a *Bedeutung*.

On the other hand, we know that as far as the sense of a sentence, the thought, is concerned, it does not matter whether the parts of the sentence have *Bedeutungen* or not.

It follows that there must be something associated with a sentence which is different from the thought, something to which it is essential that the parts of the sentence should have *Bedeutungen*.

This is to be called the *Bedeutung* of the sentence.

**But the only thing to which this is essential is what I call the truth-value—whether the thought is true or false....**

**A sentence proper is a proper name, and its *Bedeutung*, if it has one, is a truth-value: the True or the False. [Beaney 297-8]**

Michael Dummett (*Frege's Philosophy of Language*):

In speaking of sentences themselves there are two different ways in which we may regard them; and these may give rise to two distinct notions of [content]. On the one hand, we may think of sentences as complete utterances by means of which, when a specific kind of force is attached, a linguistic act may be effected: in this connection, we require that notion of [content] in terms of which the particular kind of force may be explained. On the other hand, sentences may also occur as constituent parts of other sentences, and, in this connection, may have a semantic role in helping to determine the [content] of the whole sentence: so here we shall be concerned with whatever notion of [content] is required to explain how the [content] of a complex sentence is determined from that of its components. There is no a priori reason why the two notions of [content] should coincide.

Dummett again:

If the notion of reference were introduced in the first place simply as that of the semantic role of expressions of different kinds, without an appeal to the name/bearer relation as prototype, then, at the outset, we should have no inclination to distinguish intensional from extensional contexts, or to treat the former separately; on the contrary, there would be a natural presumption in favour of a uniform semantic treatment for all contexts.

Free-standing vs. ingredient content.

Two expressions have the same ingredient content iff substituting one for the other never changes the free-standing content of expressions in which they occur.

In multivalued logic, these are designatedness values and multivalues.

Sample semantic matrix:

!	[*1]	[2]	[3]
[*1]	1	2	3
[2]	1	3	3
[3]	2	3	1

Three *multivalues*: [1], [2], and [3], of which one, [1], is *designated*, \*.

Compute the values of and consequence relations among sentences of the form ‘ $p!q$ ’.

### Kleene’s Strong Three-Valued Connective Definitions

A	$\neg A$	$A \vee B$	F	U	T	$A \& B$	F	U	T
F	T	F	F	U	T	F	F	F	F
U	U	U	U	U	T	U	F	U	U
T	F	T	T	T	T	T	F	U	T

K3 and LP (which are duals):

Using the same Strong Kleene connective definitions, get different logics by varying the definition of consequence.

K3 (Kripke):

T designated.

LP (Logic of Paradox, Graham Priest):

T and U designated.

Good consequence preserves designatedness (*salva veritate*).

K3 is a logic of truth-value *gaps*. Excluded middle does not hold. U means ‘Neither’.

$A \vee \neg A$  is not valid (does not always get a designated value), and is not implied by everything.

LP is a logic of truth-value *gluts*. Noncontradiction does not hold. U means ‘Both’.

$A \& \neg A$  is not contra-valid (can get a designated value), and does not imply everything.

These logics can be understood in terms of *consequence* rather than *truth* (result from Hlobil and Brandom [2024]):

K3 is the logic of *premissory role inclusions*:

$A |_{\sim K3} B$  iff A can be substituted for B everywhere as a premise, *salva consequentia*.

LP is the logic of *conclusory role inclusions*:

$A |_{\sim LP} B$  iff B can be substituted for A everywhere as a conclusion, *salva consequentia*.

ST (Strict-tolerant):

Consequence is *not* defined by preservation of a designated value.

Consequence holds if strictly true premises don’t lead to strictly false conclusions.

The theorems of ST are just those of classical logic.

TS is the dual of ST.

FDE (First degree entailment):

Four-valued, with extra values being ‘Neither’ (true nor false), as in K3

and ‘Both’ (true and false), as in LP.

Treat T and **Both** as designated.