Handout for Week 7

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Some of Bolzano's key definitions:

I wish to give the name of *deducibility* [***Ableitbarkeit***] to this relation between propositions *A, B, C, D,* . . . on one hand and *M, N, O,* . . . on the other.
Hence I say that propositions *M, N, O,* . . . are *deducible* from propositions *A, B, C, D,* . . . with respect to variable parts *i, j,* . . . , if every class of ideas whose substitution for *i, j,* . .. makes all of *A, B, C, D,*. . . true, also makes all of *M, N, O,* . . . true.
Occasionally, since it is customary, I shall say that propositions *M, N, O,*. . . *follow,* or can be *inferred* or *derived,* from *A, B, C, D.* . . . Propositions *A, B, C, D,... I* shall call the *premises, M, N, O,*. . . the *conclusions.* [*WL* §155]

Similar distinctions can be made with the relation of *incompatibility.* If we say of several propositions *A, B, C, D,* . . . merely that they stand in the relation of incompatibility to each other with respect to ideas *i,j, . . .,* then all we are saying is that there are no ideas whose substitution for *i, j,* . . . will make all of the propositions *A, B, C, D,* . . . true together. But by saying that *A, B, C, D,*. . . are incompatible with each other, we are not claiming that there may not be some of them, e.g. *A, B,* . . ., or *B, C,* . . . which are made true through certain common ideas, without, respectively, *C, D,* . . . or *A, B,* . . . becoming true also. [*WL* § 159]

Let us now ask whether among several incompatible propositions *A, B, C, D,* . . . and *M, N, O,* . .. there may not **be some *A, B, C,* . . . which are of such a nature that every class of ideas whose substitution for *i, j,*... makes all of them *true*, will make certain others, *M, N,0,* . . . *false*.** If this is the case, then the relation of the propositions *M, N, O,* . . . to the propositions *A, B, C,* . . . is the exact opposite of the relation which we have previously called *deducibility.*
I wish to call it the relation of *exclusion*; I shall say that one or several propositions *M, N, O,* . . . are *excluded* by certain others *A, B, C,* . . . with respect to variable ideas *i, j,* .. . if every class of ideas whose substitution for *i, j,* . . . makes all of *A, B, C,* . . . true, makes all of *M, N, O,* . . . false. *A, B, C,* . . . I call *excluding* propositions, *M, N, O.*. . . *excluded* propositions. [*WL* § 159]

3. It can also be the case that the relation of exclusion holds mutually and with respect to the same ideas i,j, . . . between propositions A, B, C, . . . and M, N, O. . . . In this case every class of ideas which make all of A, B, C, . . . true will make all of M, N, O, . . . false, and every class of ideas which will make all of M, N, O, . . . true will make all of A, B, C,. . . false. We can properly call this relation between propositions A, B, C, . . . and M, N, O, . . . a relation of mutual exclusion. [*WL* § 159]

I therefore want to give a special name to the concept of the relation of all true propositions to the total of all propositions which can be generated by treating certain ideas in a proposition as variables and replacing them with others according to a certain rule. I wish to call it the *satisfiability [Gültigkeit*] of the proposition. [*WL* § 147]

Analytic and Synthetic Propositions
1. I showed in the preceding section that there are universally satisfiable as well as non-satisfiable propositions, given that certain of their parts are considered variable. It was also shown that propositions which have either of these properties on the assumption that *i, j, . . .* are variable, do not retain this status if different or additional ideas are taken as variable. It is particularly easy to see that no proposition could be formed so as to retain such a property if *all* its ideas were considered variable. **For if we could arbitrarily vary all constituent ideas of a proposition, we could transform it into any other proposition whatever, and thus could turn it into a true, as well as a false, proposition**.
If a proposition contained even a single idea which could be arbitrarily changed without altering the truth or falsity of the proposition; i.e., the propositions which could be obtained from it through the arbitrary alteration of this one idea would either all be true or all false, provided only they have reference. Borrowing this expression from Kant, I allow myself to call propositions of this kind *analytic*. All other propositions, i.e. all those which do not contain any ideas which can be changed without altering their truth or falsity, I shall call *synthetic*. [*WL* § 148]

It follows that there are infinitely many truths ; since the assumption of any finite set of truths involves a contradiction. Suppose that somebody wants to acknowledge only *n* truths; then these truths, whatever they may be, can be represented by the following *n* formulae: *A* is *B, C* is *D,... Y* is *Z.* By claiming that only these *n* propositions should be acknowledged as true, he asserts something which could be stated in the following form: 'Aside from the propositions *A* is *B, C* is *D, . . .* Y is *Z,* no other proposition is true'. This formulation makes it evident that this proposition has entirely different parts, and therefore is different from any of the *n* propositions *'A* is *B', 'C* is *D', . . . ' Y* is *Z',* taken by themselves. Since our critic nevertheless holds this proposition to be true, he vitiates the assertion that there are only *n* true propositions, since it is the *n* + 1st. [*WL* §32]
The world is determined by the facts, and by these being *all* the facts. [*Tractatus* 1.11]