Handout for Week 3:
Grundlagen I

From *Begriffsschrift*:

§ 8. Identity of content differs from conditionality and negation in that it applies to names and not to contents. Whereas in other contexts signs are merely representatives of their content, so that every combination into which they enter expresses only a relation between their respective contents, they suddenly display their own selves **when they are combined by means of the sign for identity of content; for it expresses the circumstance that two names have the same content.** Hence the introduction of a sign for identity of content necessarily produces a bifurcation in the meaning of all signs: they stand at times for their content, at times for themselves. At first we have the impression that what we are dealing with pertains merely to the *expression* and *not to the thought,* that we do not need different signs at all for the same content and hence no sign whatsoever for identity of content. To show that this is an empty illusion I take the following· example from geometry.

Assume that on the circumference of a circle there is a fixed point *A* about which a ray revolves. When this ray passes through the center of the circle, we call the other point at which it intersects the circle the point *B* associated with this position of the ray. The point of intersection, other than *A,* of the ray and the circumference will then be called the point *B* associated with the position of the ray at any time; this point is such that continuous variations in its position must always correspond to continuous variations in the position of the ray. Hence the name *B* denotes something indeterminate so long as the corresponding position of the ray has not been specified. We can now ask: what point is associated with the position of the ray when it is perpendicular to the diameter? The answer will be: the point *A.* In this case, therefore, the name *B* has the same content as has the name *A* ; and yet we could not have used only one name from the beginning, since the justification for that is given only by the answer. **One point is determined in two ways**: (1) immediately through intuition and (2) as a point *B* associated with the ray perpendicular to the diameter.

To each of these ways of determining the point there corresponds a particular name. **Hence the need for a sign for identity of content rests upon the following consideration: the same content can be completely determined in different ways**; but that in a particular case *two ways of determining it* really yield the *same result* is the content of a *judgment.* Before this judgment can be made, **two distinct names, corresponding to the two ways of determining the content, must be assigned to what these ways determine**. The judgment, however, requires for its expression a sign for identity of content, a sign that connects these two names. From this it follows that the existence of different names for the same content is not always merely an irrelevant question of form ; rather, that there are such names is the very heart of the matter if each is associated with a different way of determining the content. In that case the judgment that has the identity of content as its object is synthetic, in the Kantian sense. A more extrinsic reason

for the introduction of a sign for identity of content is that it is at times expedient to introduce an abbreviation for a lengthy expression.
Then we must express the identity of content that obtains between the abbreviation and the original form.

Now let

$$\vdash\!\!-(A = B)$$

mean that *the sign A and the sign B have the same conceptual content, so that we can everywhere put B for A and conversely.* [§8]

In what follows, therefore, unless special notice is given, **the only 'numbers' under discussion are the positive whole numbers, which give the answer to the question "How many?".** [4]

It is no good objecting that 0 and 1 are not numbers in the same sense as 2 and 3. What answers the question "How many?" is number, and if we ask, for example, "How many moons has this planet?", we are quite as much prepared for the answer 0 or 1 as for 2 or 3, and that without having to understand the question differently. [44]

Does the number word [Zahlwort] 'one' stand for a **property** of objects?... It must strike us immediately as remarkable that every single thing should possess this property. It would be incomprehensible why we should still ascribe it expressly to a thing at all. It is only in virtue of the possibility of something not being wise that it makes sense to say "Solon is wise." The content of a concept diminishes as its extension [Umfang] increases; if its extension becomes all-embracing, its content must vanish altogether... In isolation it seems that 'one' cannot be a predicate [Praedicat]. This is even clearer if we take the plural**. Whereas we can combine "Solon was wise" and "Thales was wise" into "Solon and Thales were wise", we cannot say "Solon and Thales were one". But it is hard to see why this should be impossible, if 'one' were a property both of Solon and of Thales in the same way that 'wise' is**. [s29;p40-41]

Every attempt to define 'one' as a property having thus failed, we must finally abandon the view that in designating a thing a unit we are adding to our description of it [eine nähere Bestimmung zu sehen [34]

The symbols 1', 1", 1'" tell the tale of our embarrassment. **We must have identity--hence the 1; but we must have difference--hence the strokes**; only unfortunately, the latter undo the work of the former... It follows, therefore, that on his view there would not only be distinct ones but also distinct twos and so on; for 1""+1"" could not be substituted for [vertreten] 1"+1'". [36]

How are we to curb the arbitrariness of our ways of regarding things [die Willkühr der Auffassung**], which threatens to obliterate every distinction between one and many?** [45]

Objects	Linguistic Expressions	Activity
Decks	1: Numeral (singular term) + Sortal	Counting
Suits	4: Numeral (singular term) + Sortal	Counting
Cards	52: Numeral (singular term) +Sortal	Counting
Corners	208 Numeral (singular term) + Sortal	Counting
Numbers	Numerals (singular terms)	Counting

Why not, in fact, adopt this very apt suggestion, and **call a concept the unit relative to the Number which belongs to it?** [...Einheit zu nennen in Bezug auf die Anzahl welche ihm zukommt] [54]
Now that we have learned **that the content of a statement of number is an assertion about a concept**...[55]

For number is no whit more an object of psychology or a product of mental processes than, let us say, the North Sea is. **The objectivity [Objectivität] of the North Sea is not affected by the fact that it is a matter of our arbitrary choice which part of the water on the Earth's surface we mark off and elect to call the 'North Sea'..**. The botanist means to assert something just as factual when he gives the Number of a flower's petals as when he gives their color. The one depends on our arbitrary choice [Willkür] just as little as the other. There does, therefore, exist a certain similarity between Number and color; it consists, however, not in our becoming acquainted with them both in external things through the senses, but in their both being objective. [26]

The proposition asserting that units are isolated and indivisible can, accordingly, be formulated as follows: **Only a concept which isolates what falls under it in a definite manner, and which does not permit any arbitrary division of it into parts, can be a unit** relative to a finite Number. [54]

Sortals:
All predicates (concepts) have *circumstances of appropriate application*.
Examples: ‘red’, ‘square’, ‘clever’.
Sortal predicates (concepts) have, in addition, *criteria of identity and individuation*.
Examples: ‘dog’, ‘person’, ‘ship’, ‘biscuit’.
The notion of “same K” applies to them in their primary usage.
So they can be *counted*.

Sortal expressions are a kind of noun or *term*: *common* nouns or *general* terms.
They contrast with *singular* terms (Frege’s ‘Eigennamme’).
Mass nouns are *also* common nouns or general terms.
Examples: water, gold, (and, more speculatively, Heidegger’s ‘Dasein’ and Hegel’s ‘Geist’).
They can be *measured*, rather than *counted*, by applying units of measure: ‘pounds’, ‘liters’.

They can also be sortalized, and so generate countables: ‘bucket’, ‘piece’.

Singular terms:

When we ask someone what the number one is, or what the symbol one means [bedeute], we get as a rule the answer "Why, a **thing**". [*Grundlagen*, Introduction, p. i]

But we have already settled [festgestellt] that number words are to be understood as standing for self-subsistent objects. [dass unter den Zahlwoerten selbständige Gegenstände zu verstehen sind-- that by number words, self-subsistent objects should be understood] And that is enough to give us a class of propositions which must have a sense, namely **those which express our recognition of a number as the same again.** [der Sätze, welche ein Wiedererkennen ausdrücken.] **If we are to use a symbol a to signify [bezeichnen] an object, we must have a criterion for deciding in all cases whether b is the same as a.** [62]

Identity claims (recognition judgments, i.e. judgments expressing the recognition of an object as the same again):
When we have thus acquired a means of [a] arriving at a determinate number and of [b] recognizing it again as the same, we can assign it a number word as its proper name [zum Eigennamen geben]. [62]

Now Leibniz's definition is as follows: "Things are the same as each other, of which one can be substituted for the other without loss of truth." [Eadem sunt, quorum unum potest substitui alteri **salva veritate.**] This I propose to adopt as my own definition of identity. ...Now it is actually the case that **in universal substitutability [allgemeinen Ersetzbarkeit] all the laws of identity are contained**. [65]

To obtain the concept of Number, one must fix [feststellen] **the sense of a numerical identity** [Zahlengleichung]. [62]