Handout for Week 10

Elaborating the Metaconcepts of Semantic Content and Pragmatic Force

Course materials available at: https://spaceofreasons.netlify.app/courses/frege2025/

Plan:

1. **Two Analytic (Top-Down) Semantic Substitutional Hierarchies**:
   a) Truth and Assertion, on the side of *Bedeutung*:

Abstract contents by assimilation according to substitutional invariance *salva veritate*.

   b) Consequence and Inference, on the side of *Sinn*:

Abstract contents by assimilation according to substitutional invariance *salva consequentia*.

2. **A Bolzanian Extension of the Computational Model of Fregean Senses**:

Substitutional analysis of *multipremise* implications and incompatibilities articulating the senses of material (nonlogical) expressions, using Bolzano's definitions of reason relations.
The key: Bolzano substitutionally analyzes *implications* (consisting of premises and conclusions) as Frege substitutionally analyzes *sentences*, both to impute ingredient contents by abstraction.

3. **The Assertion/Truth species of the Force/Content distinction**.
   Lessons from a social-perspectival analysis of Justified True Belief (JTB) accounts of knowledge attributions.
   Endorsement/attribution on side of force. Anaphoric inheritance of content.

4. "The Thought":
   First-Personal and Indexical or Demonstrative Senses.
   Getting up close and personal with Dr. Gustav Lauben.
   Endorsement/attribution on side of force. Anaphoric inheritance of content.

5. "Negation":
   The Denial/Negation species of the Force/Content Distinction.
   Rejecting the construal of
   - judging-asserting as *combining* contents ('synthesis') and
   - rejecting-denying as *separating* contents.

Two Semantic Substitutional Hierarchies

Truth and **Assertion (*Bedeutung*)**:	**Consequence** and **Inference (*Sinn*):**
1. Top-level, force-relevant semantic assessment of (free-standing, i.e. unembedded) *sentences* as good or not good in the sense of *true* or *false*. This dimension of *semantic* assessment is to be understood in terms of its role in explaining or explicating *pragmatic* attitudes of *acceptance* and *rejection*, expressed in speech acts of *assertion* and *denial*.	1. Top-level, force-relevant semantic assessment of *reason relations* of *implication* and *incompatibility*, relating a set of sentences as *premises* to a sentence (or set) as *conclusion*. Candidate implications and incompatibilities can either be good or bad (hold or not hold). This dimension of *semantic* assessment is to be understood in terms of its role in explaining or explicating *pragmatic* practices of making assertions or denials of sentences giving reasons *for* (implications) and reasons *against* (incompatibilities) other assertions and denials.
2. Two sentences have the same sentential *ingredient content* iff they are intersubstitutable *salva veritate* as components of more complex sentences,: never turning a *true* compound sentence in which they occur into a *false* one.	2. Two (free-standing, i.e. unembedded) sentences have the same sentential *sense* (thought) iff they are intersubstitutable as premises and conclusions of implications and incompatibilities *salva consequentia*: never turning a good implication or incompatibility into a bad one.
3. Two essentially subsentential expressions (terms/predicates) have the same subsentential *ingredient content* iff they are intersubstitutable saving the ingredient content of sentences in which they occur.	3. Two embedded sentences have the same *ingredient sense* iff they are intersubstitutable saving the sense of the more complex sentences of which they are components. 4. Two essentially subsentential expressions (terms/predicates) have the same subsentential *ingredient sense* iff they are intersubstitutable saving the ingredient sense of sentences in which they occur.

All semantic values introduced by *abstraction* using these *substitutional equivalence relations*.

Two Connections between the Two Semantic Substitutional Hierarchies

According to the computational model:

a) Senses consist of ("contain") all the functional analyses or decompositions of the expression whose sense they are.
b) These are functions-applied-to-arguments ('f-a-t-a's).

Ingredient senses, in the consequence-and-inference substitutional hierarchy, play a dual role. The two roles are:

i. They are "modes of presentation" of the *Bedeutungen* of the senses, and
ii. They determine the *inferential role* of the expression, in the sense that two expressions have the same inferential role (and so, ingredient sense) just in case they are intersubstitutable saving the goodness of reason relations of implication and incompatibility in which the expression occurs, either as a premise or conclusion (for sentences) or as components of premises and conclusions of implication and incompatibility relations.

A) The computational model:

Re (i), modes of presentation:

All the functional analyses of an expression into f-a-t-a s yield the same *value*, when that function is applied to those arguments. So analyzing '2^4' into $\lambda y(2^y)$ applied to 4 yields the value 16, and so does analyzing '2^4'into $\lambda x(x^4)$ applied to 2, and $\lambda x,y(x^y)$ applied to <2,4>. The common value of those functions-applied-to-arguments is the *Bedeutung* of the expression. That *Bedeutung* is either the free-standing or the ingredient content of expressions, as construed in the truth-assertion substitutional hierarchy. So the functional decompositions, f-a-t-a s, that make up the senses in the consequential-inferential substitutional hierarchy are modes of presentation functionally computing the *Bedeutungen* in the truth-assertion substitutional hierarchy.

B) A Bolzanian extension of the computational model:

Re (ii), determining roles w/res to reason relations:

On the computational model, each functional decomposition of an expression into a function (including complex predicates, whose values are truth-values) and arguments (f-a-t-a) corresponds to a pattern of implications-and-incompatibilities. So we know that $2^y=2*2*..*2$, y times, and that $x^4=x*x*x*x$. Those identities license different patterns of intersubstitution inferences, with each instance of the pattern corresponding to a different argument.

But there remains a question: how do we understand the *interaction* of the senses of different sentences that is involved in *multipremise* implications of conclusions that are *not* the products of the senses of any individual premises? Frege can do this for multipremise implications (and incompatibilities) underwritten by the definitions of *logical* sentential

compounding devices (and quantifiers and identity). But he does not have a solution for the general semantic case. How does *combining* premises yield new conclusions, not contained in any of the premises? Can the method of abstracting by invariances under substitution help here?

That challenge can be responded to, and a further relation of the consequential-inferential substitutional hierarchy to the truth-assertion substitutional hierarchy made visible, if we *generalize* (and in a sense *radicalize*) the Fregean analysis by appealing to *Bolzano*'s method of deriving reason relations from substitutional analyses. Where Frege talks about 'functions', with arguments and values, Bolzano talks about dividing the vocabulary of a candidate implication or incompatibility (as a relation between sets of sentences as premises and sentences as conclusions) into two parts: a *variable* part, corresponding to Frege's *arguments*, and a *fixed* part, corresponding to Frege's *functions*.

Where Frege only applies the substitutional methodology to ***sentences*** (and their parts), Bolzano applies it at the higher level of **reason relations of *implication* and *incompatibility***.

Bolzano: **Premise-set Γ *implies* sentence A, Γ|~A, just in case every substitutional variation of the *variable* vocabulary that yields a *true* set of sentences when the fixed vocabulary of Γ is held fixed also yields a *true* conclusion when those same variable substitutions are applied to the fixed vocabulary of A** (and for incompatibility, Γ#A, if the result of varying A is *false* for true variants of Γ).

This Bolzano generalization and radicalization of the computational model of Fregean senses appeals to the semantic values *true/false* from the truth-assertion substitutional hierarchy in order to determine the *reason relations* of the consequence-inference substitutional hierarchy.

Notice that all of this discussion is *prelogical*. No mention is made of, no privilege is accorded to, specifically *logical* vocabulary in defining the two semantic substitutional hierarchies or the two kinds of connection between them (via modes of presentation and Bolzano's substitutional semantic definition of reason relations). Methods of semantic compounding of sentences or subsentential expressions *may* include logical vocabulary (conditionals, negation, quantifiers, and so on), but need not. And among the vocabulary held fixed by some Bolzanian partitions there *may* be logical vocabulary. But no privilege is accorded *semantically* to such operators or choices of functional analysis.

The suggestion is that as an alternative to understanding the relations between senses and *Bedeutungen* on the semantic model of intensions and extensions, we understand it in terms of the Bolzano version of the computational model. That is to understand the relation between the two semantic substitutional hierarchies as essentially consisting in the two kinds of relation, (A) and (B). Those relations articulate the two roles the modes of presentation contained in senses play: as modes of presentation of *Bedeutungen*, in the sense of computations yielding those *Bedeutungen* as values when a function is applied to an argument, and the *Bedeutungen* as truth values as determining *multipremise reason relations* involving the expression whose sense is in question, in the way Bolzano defines.