Frege's Mature Metavocabulary

1.   <u>Significant linguistic expressions divide exclusively and exhaustively into two species</u>:
   i.   *Complete*, and
   ii.   *Incomplete*.
Important kinds of *complete* expression include declarative *sentences* and *singular terms* (paradigmatically, *proper names*).
Incomplete expressions are all *function expressions*. They are the result of performing a certain kind of *substitutional analysis* of complete expressions, which distinguishes function-expressions from argument-expressions.
An important kind of *incomplete* expression is *predicates*, also called *concept expressions*.

2.   *<u>Sinn/Bedeutung</u>*:
   a) All properly used or introduced linguistic expressions of either kind (complete or incomplete) *express* a *sense* (*Sinn*).
Senses are what they are linguistic expressions *of*.
   b) A distinguished and privileged subset of *senses designate* (*bedeuten*) or stand for something: a reference or referent (*Bedeutung*). They all 'purport' to do so.
      "A proper name ([or, more generally, so as to include also *in*complete expressions, cf. "Comments on USB" (BB)] word, sign, sign combination, expression) *expresses* its sense, *stands for* or *designates* its reference.
      By means of a sign we express its sense and designate its reference." [USB 61]
Note that strictly it is only *senses* that directly designate or have *Bedeutungen*.
Signs are associated with *Bedeutungen* only indirectly, by and insofar as they *express senses*, that is, insofar as they are linguistic *expressions* (of senses).

3.   <u>(Declarative) Sentences</u>:
   a) The sense of a sentence is called a *thought*—in the sense of a thinkable, something that is thought, not the act of thinking it.
   b) Sentences are what can be used to *assert* or *deny*.
      Asserting is taking-true and denying is taking-false.
      So thoughts, expressed by sentences as their senses, are what can be assessed as true or false.
   c) In addition to being *taken* to be true or false, thoughts can *be* true or false.
   d) In the 1879 *Begriffsschrift*, declarative sentences are what can play the role of *premises* and *conclusions* of *implications*, or relations of *consequence*.

4.   Truth-values (*Wahrheitswerte*):

    a) Truth-values are introduced by *abstraction*.
(Compare: the truth-value of a thought and the direction of a line.)
$\mathrm{TV}(p) = \mathrm{TV}(q) \iff p \approx q$, where '$\approx$' is the equivalence relation that holds between p and q just in case either p is true and q is true, or p is false and q is false. These are both reflexive, symmetric, and transitive relations. So thoughts expressed as the senses of sentences are sorted into two groups: all the true ones have the same truth-value, and all the false ones have the same truth-value. The two truth-values introduced in this way by abstraction can be given canonical proper names 'the True' and 'the False.'
    b) Truth-values are the *Bedeutungen* of the thoughts (that is, the *senses*) expressed by
        sentences.
    c) Predicates or concept-expressions are function-expressions whose values, when applied
        to arguments, are truth-values.

5.   Two principles regarding *Bedeutungen* in general:

    i.   <u>*Bedeutungen* are introduced by abstraction</u>:
$\mathrm{BD}(t) = \mathrm{BD}(t') \iff \mathrm{TV}(F(t)) = \mathrm{TV}(F(t'))$, for all predicates or concept-expressions F.
(Compare: the *Bedeutung* of an expression and the direction of a line.)
That is, uniformly substituting t for t', or t' for t, never turns a true thought into a thought that is not true.   The expressions t and t' are intersubstitutable *salva veritate*.

    ii.   <u>Name/bearer model</u>:
"If words are used in the ordinary way, what one intends to speak of is their reference." [USB58]
"The reference of a proper name is the object itself which we designate by its means." [USB 60]
That is: **$\mathrm{BD}(t) = t$**.

6.   Two principles regarding *senses* in general:

    i.   <u>Senses are introduced by abstraction.</u>
Frege introduces the concept of *cognitive-value* (*Erkenntnis*werte), by analogy in the dimension of *sense* to *truth*-values (*Wahrheits*werte) in the dimension of *Bedeutung*.
    Sense : *Cognitive*-value :: *Bedeutung* : *Truth*-value.
$\mathrm{Sense}(a) = \mathrm{Sense}(b) \iff \mathrm{CV}(F(t)) = \mathrm{CV}(F(t'))$, for all predicates or concept-expressions F,
That is, uniformly substituting t for t', or t' for t, never changes the *cognitive value* of the thoughts expressed. Different cognitive values means different thoughts (senses).

Question: **What are *cognitive values*?**

ii.   Senses designate *Bedeutungen* by "containing modes of presentation" (*Art des Gegebenseins*) of those *Bedeutungen*:

"It is natural, now, to think of there being connected with a sign (name, combination of words, letter), besides that to which the sign refers, which may be called the reference of the sign, also what I should like to call **the *sense* of the sign, wherein the mode of presentation is contained**… The reference of' evening star' would be the same as that of 'morning star,' but not the sense. [USB 57]

Question: **What are *modes of presentation* of *Bedeutungen*?**

7.   An interpretive hypothesis:

a) "When I wrote my *Grundlagen der Arithmetik,* I had not yet made the distinction between sense and reference and so, under the expression 'a possible content of judgment,' I was combining what I now designate by the distinctive words 'thought' and 'truth-value.' " [CO 47]

b) Throughout his work, Frege employs the same fundamental method: Assimilating linguistic expressions accordingly as *intersubstitution* preserves some higher-level property, and then using the resulting equivalence relation to introduce new sortal kind-terms and singular terms falling under them whose criteria of identity and individuation are determined by *abstraction*.

     i.   In the 1879 *Begriffsschrift*, two sentences are said to have or express the same *conceptual content* (*begrifflich Inhalt*) (compare: two lines have the same direction) just in case substituting one for the other as the premise of an implication never turns a good implication (one whose conclusion follows from or is a consequence of its premises) into a bad one.

     ii.   In the 1891-2 essays, the senses of two expressions determine, designate, or present the same *Bedeutung* just in case substituting one for the other as the argument of a predicate-function never turns a *true* thought into one that is not true.

c) Those two bits of semantic abstraction by noting invariance under substitution assimilate linguistic expressions accordingly as they are intersubstitutable *salva consequentia* (to abstract *begriffliche Inhalte*), or *salva veritate* (to abstract *Bedeutungen*).

d) Conjecture: Frege came to distinguish *sense* from *Bedeutung* by distinguishing the results of abstraction according to intersubstitutability *salva consequentia* from the results of abstraction according to intersubstitutability *salva veritate*.

Senses : intersubstitutability *salva consequentia*

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Bedeutungen : intersubstitutability *salva veritate*.

8.   Challenges for such an understanding of the metaconcept *Sinn*:

a) How to understand the notion of <u>cognitive value</u>, which is officially what is preserved by intersubstitution of expressions with the same sense, in terms of <u>inferential role</u>, which is what the proposal suggests is preserved by intersubstitution of expressions with the same sense.

b) How to understand the claim that senses *contain* <u>modes of presentation</u> of *Bedeutungen*, in terms of senses as inferential roles. This is the claim that is often formulated less carefully as "sense determines reference."

9.   Challenges for such an understanding of the metaconcept *Bedeutung*:

a) How to understand the role of the *name/bearer model* in relation to intersubstitutability *salva veritate*.

b) In particular, how are we to understand the claimed *identity* of the *Bedeutungen* of ordinary singular terms falling under *concrete* sortal kind-terms referring to perceptible objects, such as 'Fido' referring to the dog, on the one hand, and singular terms falling under sortal kind-terms introduced by *abstraction*, such as "the *Bedeutung* of 'Fido'"? How can it be that *Bedeutung*('Fido') = Fido ?
Compare: Can England, or Julius Caesar, *be* (be identical to or with) the direction of the Earth's axis?