Passages from “Boole’s Logical Calculus and the Concept-Script” [1880-81]

In *Posthumous Writings,* Hans Hermes, Friedrich Kambartel, Friedrich Kaulbach (eds.) [Blackwell, Oxford, 1979].

…an idea which Leibniz clung to throughout his life with the utmost tenacity, the idea of a *lingua characterica,* an idea which in his mind had the closest possible links with that of a *calculus ratiocinatur.* That it made it possible to perform a type of computation, it was precisely this fact that Leibniz saw as a principal advantage of a script which compounded a concept out of its constituents rather than a word out of its sounds, and of all hopes he cherished in this matter, we can even today share this one with complete confidence. [9]

In a short monograph, I have now attempted a fresh approach to the Leibnizian idea of a *lingua characterica.* [10]

In contrast we may now set out the aim of my concept-script. Right from the start I had in mind the *expression of a content.* What I am striving after is a *lingua characterica* in the first instance for mathematics, not a *calculus* restricted to pure logic. But the content is to be rendered more exactly than is done by verbal language. [12]

The reason for this inability to form concepts in a scientific manner lies in the lack of one of the two components of which every highly developed language must consist. That is, we may distinguish the formal part which in verbal language comprises endings, prefixes, suffixes and auxiliary words, from the material part proper. The signs of arithmetic correspond to the latter. What we still lack is the logical cement that will bind these building stones firmly together….In contrast, Boole's symbolic logic only represents the formal part of language, and even that incompletely. [13]

Thus, the problem arises of devising signs for logical relations that are suitable for incorporation into the formula language of mathematics, and in this way of forming-at least for a certain domain-a complete concept-script. This is where my booklet comes in. [14]

So it transpires that even when we restrict ourselves to pure logic my concept-script commands a somewhat wider domain than Boole's formula language. This is a result of my having departed further from Aristotelian logic. For in Aristotle, as in Boole, the logically primitive activity is the formation of concepts by abstraction, and judgement and inference enter in through an immediate or indirect comparison of concepts via their extensions. [15]

As opposed to this, I start out from judgements and their contents, and not from concepts. The precisely defined hypothetical relation between contents of possible judgement has a similar significance for the foundation of my concept-script to that which identity of extensions has for Boolean logic. I only allow the formation of concepts to proceed from judgements. [16]

If, that is, you imagine the 2 in the content of possible judgement

$2^4 = 16$

to be replaceable by something else, by (-2) or by 3 say, which may be indicated by putting an *x* in place of the 2:

$x^4 = 16$,

the content of possible judgement is thus split into a constant and a variable part. The former, regarded in its own right but holding a place open for the latter, gives the concept '4th root of 16'. We may now express by the sentences '2 is a fourth root of 16' or 'the individual 2 falls under the concept "4th root of 16"' or 'belongs to the class of 4th roots of 16'. But we may also just as well say '4 is a logarithm of 16 to the base 2'. Here the 4 is being treated as replaceable and so we get the concept 'logarithm of 16 to the base 2':

$2^x=16$.

the *x* indicates here the place to be occupied by the sign for the individual falling under the concept. We may now also regard the 16 in $x^4 = 16$ as replaceable in its turn, which we may represent, say, by $x^4 = y$. In this way we arrive at the concept of a relation, namely the relation of a number to its 4th power. And so instead of putting a judgement together out of an individual as subject and an already previously formed concept as predicate, we do the opposite and arrive at a concept by splitting up the content of possible judgement. [17]

But the usefulness of such formulae only fully emerges when they are used in working out inferences, and we can only fully appreciate their value in this regard with practice. [27]

In the preface of my *Begriffsschrift* I already said that the restriction to a single rule of inference which I there laid down was to be dropped in later developments. This is achieved by converting what was expressed as a judgement in a formula into a rule of inference. I do this with formulae (52) and (53) of the *Begriffsschrift,* whose content I render by the rule: in any judgement you may replace one symbol by another, if you add as a condition the equation between the two. [29]

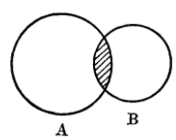
Precision and rigour are the prime aims of the concept-script; brevity will only be sought after if it can be achieved without jeopardizing those aims. [32]

I now return once more to the examples mentioned earlier, so as to point out the sort of concept formation that is to be seen in those accounts. The fourth example gives us the concept of a multiple of 4, if we imagine the 12

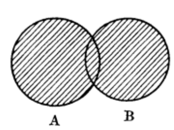
$$\text{in } \frac{\gamma}{\beta}\,(0_\gamma + 4 = 12_\beta):$$

as replaceable by something else; the concept of the relation of a number of its multiple if we imagine the 4 as also replaceable; and the concept of a factor of 12 if we imagine the 4 alone as replaceable. The 8th example gives us the concept of the congruence of two numbers with respect to a modulus, the 13th that of the continuity of a function at a point etc. All these concepts have been developed in science and have proved their fruitfulness. For this reason what we may discover in them has a far higher claim on our attention than anything that our everyday trains of thought might offer. For fruitfulness is the acid test of concepts, and scientific workshops the true field of study for logic. [32-3]

Now it is worth noting in all this, that in practically none of these examples is there first cited the genus or class to which the things falling under the concept belong and then the characteristic mark of the concept, as when you define 'homo' as *"animal rationale'.* Leibniz has already noted that here we may also conversely construe *'rationale'* as genus and *'animal'* as species. In fact, by this definition *'homo'* is to be whatever is *'animal'* as well as being *·rationale'.*



If the circle *A* represents the extension of the concept *'animal'* and *B* that of *'rationale',* then the region common to the two circles corresponds to the extension of the concept *·homo'.* And it is all one whether I think of that as having been formed from the circle *A* by its intersection with *B* or vice versa. This construction corresponds to logical multiplication. Boole would express this, say, in the form $C = AB$, where *C* means the extension of the concept *'homo'.* You may also form concepts by logical addition.



We have an example of this if we define the concept 'capital offence' as murder or the attempted murder of the Kaiser or of the ruler of one's own *Land* or of a German prince in his own *Land.* The area *A* signifies the extension of the concept 'murder', the area *B* that of the concept 'attempted murder of the Kaiser or of the ruler of one's own *Land* or of a German prince in his own *Land'.* Then the whole area of the two circles, whether they have a region in common or not, will represent the extension of the concept 'capital offence'.
If we look at what we have in the diagrams, we notice that in both cases the boundary of the concept, whether it is one formed by logical multiplication or addition is made up of parts of the boundaries of the concepts already given. This holds for any concept formation that can be represented by the Boolean notation. This feature of the diagrams is naturally an expression of

something inherent in the situation itself, but which is hard to express without recourse to imagery. In this sort of concept formation, one must, then, assume as given a system of concepts, or speaking metaphorically, a network of lines. These really already contain the new concepts: all one has to do is to use the lines that are already there to demarcate complete surface areas in a new way. It is the fact that attention is principally given to this sort of formation of new concepts from old ones, while other more fruitful ones are neglected which surely is responsible for the impression one easily gets in logic that for all our to-ing and fro-ing we never really leave the same spot. [33-4]

If we compare what we have here with the definitions contained in our examples, of the continuity of a function and of a limit, and again that of following a series which I gave in § 26 of my *Begriffsschrift,* we see that there's no question there of using the boundary lines of concepts we already have to form the boundaries of the new ones. Rather, totally new boundary lines are drawn by such definitions-and these are the scientifically fruitful ones. Here too, we use old concepts to construct new ones, but in so doing we combine the old ones together in a variety of ways by means of the signs for generality, negation and the conditional. [34]

I believe almost all errors made in inference to have their roots in the imperfection of the concepts. Boole presupposes logically perfect concepts as ready to hand, and hence the most difficult part of the task as having been already discharged; he can then draw his inferences from the given assumptions by a mechanical process of computation. Stanley Jevons has in fact invented a machine to do this. But if we have perfect concepts whose content we do not need to refer back to, we can easily guard ourselves from error, even without computation. [34-5]

But we can only derive any real benefit from doing this, if the content is not just indicated but is constructed out of its constituents by means of the same logical signs as are used in the computation. [35]

But it is a basic principle of science to reduce the number of axioms to the fewest possible. Indeed the essence of explanation lies precisely in the fact that a wide, possibly unsurveyable, manifold is governed by one or a few sentences. The value of an explanation can be directly measured by this condensation and simplification: it is zero if the number of assumptions is as great as the number of facts to be explained. [36]

I believe in this essay I have shown:
(1) My concept-script has a more far-reaching aim than Boolean logic, in that it strives to make it possible to present a content when combined with arithmetical and geometrical signs.
(2) Disregarding content, within the domain of pure logic it also, thanks to the notation for generality, commands a somewhat wider domain than Boole's formula-language.

(3) It avoids the division in Boolean logic into two parts (primary and secondary propositions) by construing judgements as prior to concept formation.
(4) It is in a position to represent the formations of the concepts actually needed in science, in contrast to the relatively sterile multiplicative and additive combinations we find in Boole.
(5) It needs fewer primitive signs for logical relations and hence fewer primitive laws.
(6) It can be used to solve the sort of problems Boole tackles, and even do so with fewer preliminary rules for computation. This is the point to which I attach least importance, since such problems will seldom, if ever, occur in science. [46]