Now in considering the question of to which of these two kinds arithmetical judgements belong, I first had to see how far one could get in arithmetic by inferences alone, supported only by the laws of thought that transcend all particulars. The course I took was first to seek to reduce the concept of ordering in a series to that of *logical* consequence, in order then to progress to the concept of number. [Preface, Beaney p. 48.]

The expression of anything that is without significance for *logical inference* has therefore been eschewed. I have called, in §3, that which solely mattered to me *conceptual content [begriffiicher Inhalt].* This point must therefore always be kept in mind if the nature of my formula language is to be understood correctly. From this the name 'Begriffsschrift' also arose. [Preface, Beaney p. 49.]

I believe I can make the relationship of my *Begriffsschrift* to ordinary language clearest if I compare it to that of the microscope to the eye. The latter, due to the range of its applicability, due to the flexibility with which it is able to adapt to the most diverse circumstances, has a great superiority over the microscope. Considered as an optical instrument, it admittedly reveals many imperfections, which usually remain unnoticed only because of its intimate connection with mental life. But as soon as scientific purposes place great demands on sharpness of resolution, the eye turns out to be inadequate. The microscope, on the other hand, is perfectly suited for just such purposes, but precisely because of this is useless for all others. [Preface, Beaney p. 49.]

Leibniz too recognized - perhaps overestimated - the advantages of an appropriate symbolism. His conception of a universal characteristic, a *calculus philosophicus* or *ratiocinator,* B was too grandiose for the attempt to realize it to go further than the bare preliminaries.
[Preface, Beaney p. 49.]

I am convinced that my *Begriffsschrift* can be successfully applied wherever a special value has to be placed on the validity of proof, as in the case of laying the foundations of the differential and integral calculus.
It seems to me to be even easier to extend the domain of this formula language to geometry. Only a few more symbols would have to be added for the intuitive relations that occur here. In this way one would obtain a kind of *analysis situs.*
The transition to the pure theory of motion and thence to mechanics and physics might follow here. In the latter fields, where besides conceptual necessity, natural necessity prevails, a further development of the symbolism with the advancement of knowledge is easiest to foresee.
But that is no reason to wait until such advancement appears to have come to an end.
[Preface, Beaney p. 50.]

If it is a task of philosophy to break the power of words over the human mind, by uncovering illusions that through the use of language often almost unavoidably arise concerning the relations of concepts, by freeing thought from I the taint of ordinary linguistic means of expression, then my *Begriffsschrift,* further developed for these purposes, can become a useful tool for philosophers. [Preface, Beaney p. 50-1.]

In particular, I believe that the replacement of the concepts *subject* and *predicate* by *argument* and *function* will prove itself in the long run. It is easy to see how taking a content as a function of an argument gives rise to concept formation. What also deserves notice is the demonstration of the connection between the meanings of the words: if, and, not, or, there is, some, all, etc. [Preface, Beaney p. 50-1.]

§2. A judgement will always be expressed by means of the symbol |— which stands to the left of the symbol or complex of symbols which gives the content of the judgement. If the small vertical stroke at the left end of the horizontal one I is *omitted,* then the judgement will be transformed into a *mere complex of ideas,* of which the writer does not state whether he recognizes its truth or not…
The horizontal stroke may be called the *content stroke,* the vertical the *judgement stroke.* The content stroke serves generally to relate any symbol to the whole formed by the symbols that follow the stroke. *What follows the content stroke must always have a judgeable content.*

§3 …the contents of two judgements can differ in two ways: **either the conclusions that can be drawn from one when combined with certain others I also always follow from the second when combined with the same judgements or else this is not the case**. The two propositions 'At Plataea the Greeks defeated the Persians' and 'At Plataea the Persians were defeated by the Greeks' differ in the first way. Even if a slight difference in sense can be discerned, the agreement predominates. Now **I call that part of the content that is the *same* in both the *conceptual content*… [O]*nly this* has significance for the *Begriffsschrift*…**

[I]*n my formula language…the only thing that is relevant in a judgement is that which influences its possible consequences. Everything that is necessary for a valid inference is fully expressed; but what is not necessary is mostly not even indicated; nothing is left to guessing.* [§3]

A distinction is drawn between *universal* and *particular* judgements: **this is not really a distinction between judgements, but between contents**. *One ought to say: 'a judgement with a universal content', 'a judgement with a particular content'.* For these properties belong to the content even when it is *not* presented as a judgement, but as a proposition. (See §2.) The same applies to negation. In an indirect way one says, for example: 'Suppose that the lines *AB* and *CD* were not equal'. Here the content, that the lines *AB* and *CD* are not equal, contains a negation,

but this content, although capable of being judged, is nevertheless not presented as a judgement. Negation therefore attaches to the content… [§4]

If I call a proposition necessary, I thereby give a hint as to my grounds for judgement. *But since this does not affect the conceptual content of the judgement, the apodeictic form of a judgement has no significance for us.* [§4]

Conditionality
§5. If *A* and *B* denote14 judgeable contents (§2), then there are the following four possibilities:
(1) *A* is affirmed and *B* is affirmed;
(2) *A* is affirmed and *B* is denied;
(3) *A* is denied and *B* is affirmed;
(4) *A* is denied and *B* is denied.
now denotes the judgement that *the third of these possibilities does not obtain, but one of the other three does.* [§4]

The causal link implicit in the word 'if', however, is not expressed by our symbols, although a judgement of this kind can be made only on the basis of such a link. For this link is something general, but an expression for generality has not yet been introduced (see §12). [§4]

In logic, following Aristotle, a whole series of modes of inference are enumerated; I use just this one [*modus ponens*]- at least in all cases where a new judgement is derived from more than one single judgement. [§6]
For the truth that lies in another mode of inference can be expressed in a judgement of the form: if *M* holds, and if *N* holds, then *A* holds too; in symbols:
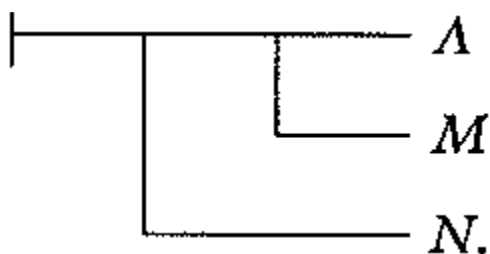
From this judgement and |—*N* and |—*M,* |—*A* follows as above. Thus an inference of whatever kind can be reduced to our case. Since it is therefore possible to manage with a single mode of inference, perspicuity demands that this be done. [§6]

Instead of expressing 'and' by means of the symbols for conditionality and negation, as is done here, conditionality could also be represented, conversely, by means of a symbol for 'and' and the symbol for negation. [§7]

The need for a symbol for identity of content thus rests on the following: the same content can be fully determined in different ways; but that, in a particular case, *the same content* is actually given by *two modes of determination* is the content of a *judgement.* Before this judgement can be made, two different names corresponding to the two modes of determination must be provided for that that is thereby determined. But the judgement requires for its expression a symbol for I identity of content to combine the two names. It follows from this that different names for the same content are not always merely a trivial matter of formulation, but touch the very heart of the matter if they are connected with different modes of determination. [§8]

If, in an expression (whose content need not be a judgeable content), a simple or complex symbol occurs in one or more places, and we think of it as replaceable at all or some of its occurrences by another symbol (but everywhere by the same symbol), then we call the part of the expression that on this occasion appears invariant the function, and the replaceable part its argument. [§9]

For us the different ways in which the same conceptual content can be taken as a function of this or that argument has no importance so long as function and argument are fully determined. But if the argument becomes *indeterminate* as in the judgement 'You can take as argument for "being representable as the sum of four squares" whatever positive whole number you like: the proposition always remains correct', then the distinction between function and argument acquires significance with regard to *content.* [§9]

One sees here particularly clearly that the concept of function in Analysis, which in general I have followed, is far more restricted than that developed here. . [§10]

§11. In the expression of a judgement, the complex of symbols to the right of |— can always be regarded as a function of one of the symbols occurring in it. *If a Gothic [old German] lerrer is put in place of the argument, and a concavity containing this letter inserted in the content stroke, as in*
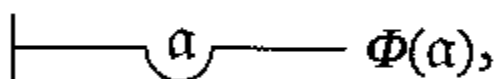
then this signifies the judgement that the function is a fact whatever may be taken as its argument.

All other conditions which must be imposed on what may replace a Gothic letter are to be included in the judgement. From such a judgement, therefore, however many *judgements with less general content* we like can be derived, by replacing the Gothic letter each time by something different, the concavity in the content stroke disappearing again. [§11]